THE GROWTH AND DECAY OF WEAK DISCONTINUITIES IN RELATIVISTIC FLUIDS WITH VIBRATIONAL RELAXATION

By

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The propagation of weak discontinuities in relativistic fluids with vibrational relaxation has been studied. The velocity of propagation of a relativistic weak discontinuity has been determined. The fundamental equation governing the growth and decay of a relativistic weak wave has been obtained and solved. The relativistic results are shown to be in full agreement with earlier results of classical gasdynamics. The problem of breakdown of weak discontinuities has also been investigated. The critical time \( t_c \) is determined when the breakdown of the wave will occur and consequently a shock wave will be formed due to non-linear steepening. It is shown that there exists a critical amplitude of the wave such that all compressive waves with an initial amplitude greater than the critical one will break down and a shock-type discontinuity will be formed, while an initial amplitude less than the critical one will result in a decay of the wave. The local and global behaviour of the wave amplitude is also discussed.

I. Introduction

Weak waves have been extensively studied during the last decade. BECKER [1] and BOWEN and CHEN [2] studied various properties of acceleration waves in non-equilibrium flows. RARITY [3] studied the problem of breakdown of acceleration waves in flows with vibrational relaxation.

The recent advances in space technology have drawn a great deal of attention towards the study of wave propagation in relativistic gasdynamics. ECKART [4] and TAUB [5] provided theoretical foundations of relativistic shock waves. The relativistic theory of propagation of weak waves in a perfect gas has been treated by SAINI [6], COBURN [7] and KANWAL [8]. The growth of weak waves in relativistic gasdynamics has also been studied by McCARTHY [9] for an ideal perfect gas. Nonequilibrium effects on the breakdown of weak waves have been recently studied by RAM [10]. The main academic interest of the present paper is to study the problem of growth and decay of relativistic weak waves in gas flows with vibrational relaxation and to determine a critical stage when there occurs a breakdown of the weak wave and the consequent formation of a shock wave.
II. Basic preliminaries

The notations used in this paper are, with a few minor exceptions, identical with those employed by Grot and Eringen [11].

Let $X^k$ be the rectangular coordinates of a material point in a three dimensional space. The motion of a material body can be described by a new set of coordinates $x^k$ given by

$$x^k = x^k(X^i, x^4); \quad x^4 = ct, \quad (i, k = 1, 2, 3),$$

where $t$ is the time and $c$ is the constant velocity of light in vacuum. Let us introduce the concept of an Einstein—Riemann space $V_4$ by four coordinates $x^* = (x^k, x^4)$ with a metric $ds^2 = \Gamma_{\alpha\beta} dx^\alpha dx^\beta$. The metric has constant components given by

$$\Gamma^\alpha\beta = \Gamma_{\alpha\beta};$$
$$\Gamma^{ij} = \Gamma_{ij} = \delta_{ij}, \quad \Gamma^{44} = \Gamma'_4 = -1.$$

The world velocity can be expressed as

$$U^\alpha(x^4) = \beta \left( \frac{v^k}{c}, 1 \right), \quad U^\alpha U^\alpha = -1;$$

where

$$v^k = c \frac{\partial x^k}{\partial x^4}, \quad \beta = (1 - v^2/c^2)^{-1/2}.$$  \hspace{1cm} (2.2)

The equations of motion of a gas with vibrational relaxation in a relativistically correct form can be written in the form:

$$\left( q U^\alpha \right)_\alpha = 0$$

$$T_{\alpha\beta} = 0,$$

$$Dq - \frac{\beta}{c} q \Phi(q - q) = 0,$$  \hspace{1cm} (2.6)