REMARK ON THE $\pi^0 \rightarrow 3\gamma$ DECAY

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A theoretical upper limit on the rate of the C violating decay is given.

One of the discussed possibilities of (electromagnetic) C violation is the possible $\pi^0 \rightarrow 3\gamma$ decay [1]. Our aim was to find the simplest $\pi^0 \rightarrow 3\gamma$ vertices and to estimate the expected branching ratio

$$R = \frac{\Gamma(\pi^0 \rightarrow 3\gamma)}{\Gamma(\pi^0 \rightarrow 2\gamma)}$$

for the case of an electromagnetic (or strong) C violation. Lorentz symmetry, $P$ conservation, Bose statistics and gauge invariance for photons are assumed. It has been shown that all the vertices with six or less derivates vanish for $0^- \rightarrow 3\gamma$ (and also for $0^+ \rightarrow 3\gamma$) transition. Independent non-vanishing vertices with seven derivates are

\begin{align*}
1. & \frac{e^3}{m^7} \partial_\xi \partial_\phi \partial_\xi F_{e\phi} \partial_\gamma F_{\xi\beta} F_{\gamma\delta}, \\
2. & \frac{e^3}{m^7} \partial_\gamma \partial_\nu \partial_\xi F_{e\nu} D F_{\gamma\nu} \partial_\xi F_{\mu\nu}, \\
3. & \frac{e^3}{m^7} \partial_\nu \partial_\mu \partial_\xi F_{e\mu} D F_{\nu\mu} \partial_\xi F_{\mu\nu}, \\
4. & \frac{e^3}{m^7} \partial_\alpha \partial_\beta \partial_\xi F_{e\beta} D F_{\gamma\beta} \partial_\xi F_{\gamma\delta}, \\
5. & \frac{e^3}{m^7} \partial_\beta \partial_\gamma \partial_\xi F_{e\beta} D F_{\rho\gamma} \partial_\xi F_{\gamma\delta}, \\
6. & \frac{e^3}{m^7} \partial_\delta \partial_\gamma \partial_\xi F_{e\beta} D F_{\gamma\delta} \partial_\xi F_{\gamma\delta}.
\end{align*}

Here $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ is the electromagnetic field tensor,

$$F_{\mu\nu}^{D} = \varepsilon_{\nu\rho\sigma} F_{\rho\sigma}$$
the corresponding pseudotensor, $m^{-1}$ is a constant of the dimension of length (the so-called decay length). There are several other independent vertices where the antisymmetric pseudotensor $\epsilon_{\mu\nu\rho}$ and the field tensor $F_{a\beta}$ are not contracted twice, so the vertex cannot be expressed in terms of $\pi^0$, $F_{a\beta}$, $F_{a\beta}^D$ only.

One of the simplest cases seems to be the vertex (1) which was quoted already by S. Barshay [2]. For a pion at rest in three dimensional language it can be rewritten in the following form:

$$\frac{e^3 m^2_{\pi}}{m^7} \pi^0 \left[ (H_2 k_1) \left\{ [E_3 H_1 k_2] + [H_3 E_1 k_2] \right\} + (H_3 k_2) \left\{ [E_1 H_2 k_3] + [H_1 E_2 k_3] \right\} + (H_1 k_3) \left\{ [E_2 H_3 k_1] + [H_2 E_3 k_1] \right\} \right].$$

The decay rate corresponding to the vertex (1a) can be expressed in a straightforward way. By making use of the $\delta$ functions several integrals can be evaluated directly; finally two integrals survive (with respect to the two photon energies $k_1$ and $k_2$). The integration domain is the central solid triangle of the Dalitz-diagram (Fig. 1). The numerical evaluation of this integral results in the estimation

$$\Gamma(\pi^0 \rightarrow 3\gamma) = 2 \cdot 10^9 \text{ s}^{-1}$$

if the decay length $m^{-1}$ is put equal to the pion Compton wavelength. This corresponds to a branching ratio

$$R = 3 \cdot 10^{-7}.$$

This numerical value may be considered to be an upper limit. A more realistic result can be obtained even in the case of maximum electromagnetic $C$ violation by putting the $\varphi$ meson Compton wavelength as decay length ($m = m_{\varphi}$):

$$R \sim 10^{-12}.$$