PROPAGATION OF A HYDROMAGNETIC SHOCK WAVE IN A STEADY FLOW OF AN IDEAL DISSOCIATING GAS

By

V. D. SHARMA and RISHI RAM

APPLIED MATHEMATICS SECTION, INSTITUTE OF TECHNOLOGY, BANARAS HINDU UNIVERSITY, INDIA

(Received 5. IX. 1975)

The propagation of a hydromagnetic shock wave in steady flow of an ideal dissociating gas is discussed. The variations of flow and field parameters along the direction of propagation of the shock are determined. It is shown that the effects of dissociation on the generation of vorticity and current density vanish, if the magnetic field is applied in the direction of propagation of the shock.

1. Introduction

In dealing with problems of hypersonic flight at high altitude a temperature of many thousand degrees of Kelvin can easily be attained during flight as the kinetic energy of the re-entering craft is dissipated by the atmospheric gas through shock compression and viscous heating. The air molecules, atoms and other species which absorb this kinetic energy may go through a change of chemical composition. In the temperature range from 1000 °K to 7000 °K the only chemical reaction involved is that of dissociation and thus the ionization and electronic excitation may be neglected. The problem of incorporating the effects of the large energy change involved in dissociation into the standard theory of gas flow appears at the same time so important and so formidable that it is worth approaching slowly. One can usefully begin, on both the theoretical and experimental sides, by eliminating the less essential complications which arise from the detailed composition of air, and studying the dynamics of a pure dissociating diatomic gas. Thus, the present work is confined to the study of the effects of non-equilibrium in molecular dissociation and atomic recombination in an inviscid flow problem only. The following assumptions are made:

(i) The molecular transport effects leading to viscosity, diffusion and heat conduction of the gases are neglected;
(ii) A diatomic gas mixture is assumed and each component of the reacting mixture is assumed thermally perfect;
(iii) In the temperature range 1000 °K \( \sim \) 7000 °K, for diatomic gases, the contributions of energy from electronic excitation and ionization are both assumed negligible,
At temperatures where dissociation is important, the radiation heat loss from the gas mixture may not be negligibly small. However, in order to simplify the problem, we exclude such an effect.

LIGHTHILL [1] introduced a new gas model termed as 'ideal dissociating gas' and deduced the oblique shock wave relations in the strong shock approximation. Considering an ideal dissociating gas and a rate equation of his own, FREEMAN [2] studied some of the major features of non-equilibrium flows past a blunt body. EPSTEIN [3] studied the problem of non-equilibrium dissociative flow behind a plane oblique shock. CAPIAUX and WASHINGTON [4] have investigated the non-equilibrium dissociative flow past a wedge by assuming the gas to be LIGHTHILL's dissociating one. HSU [5] determined the flow gradients behind a curved shock in dissociating gases. HSU and ANDERSON [6] obtained the variations of pressure, density, temperature and degree of dissociation along the stream lines in a non-equilibrium flow of a diatomic gas behind a plane oblique shock. In this paper keeping in mind the aforementioned assumptions (i)—(iv), we shall study the down-stream effects of the propagation of a hydromagnetic shock in a steady flow of an ideal dissociating gas. For simplicity we shall assume the upstream flow to be uniform and frozen.

2. Basic equations

For a mixture of perfect gases, the continuity equation of the $i^{th}$ species is [7]

$$\rho (u \cdot \nabla) x_i = \sigma_i,$$  \hspace{1cm} (2.1)

where $\rho$ and $u$ denote the density and velocity vector for the mixture. $\sigma_i$ denotes the mass rate of production of species $i$ per unit volume by chemical reaction and $x_i \equiv \rho_i/\rho$ is the mass fraction of the $i^{th}$ species. The magnetogasdynamic flow equations for the mixture are

$$\rho (\nabla \cdot u) + u \cdot \nabla \rho = 0,$$ \hspace{1cm} (2.2)

$$\rho (u \cdot \nabla) u + \nabla p + \frac{1}{4\pi} H \times (\nabla \times H) = 0,$$ \hspace{1cm} (2.3)

$$(u \cdot \nabla) H - (H \cdot \nabla) u + H (\nabla \cdot u) = 0,$$ \hspace{1cm} (2.4)

$$\rho u \cdot \nabla h - u \cdot \nabla \rho = 0,$$ \hspace{1cm} (2.5)

where $p$, $H$ and $h$ are respectively the pressure, magnetic field vector and the enthalpy which for the dissociating gas mixture is given by a functional relation of the form:

$$h = h(p, \rho, x_i).$$  \hspace{1cm} (2.6)