STRONG INTERACTIONS IN THE FOUR-DIMENSIONAL ISOTOPIC SPACE

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A new variant of the mathematical scheme in the four-dimensional isotopic spin space will be proposed for the particles involved in strong interactions, which is suited to give a review of the relations found by GELL-MANN and others. Nucleons, hyperons of order one and hyperons of order two will be described by spinor quantities $B_\alpha$, $B_{\alpha\dot{\alpha}}$ and $B_{\alpha\dot{\alpha}\gamma}$ respectively, $\tau$- and $K$-mesons by the spinors $\eta^\nu$ and $K^\tau$. A possible connection with the GOLDHABER—GYÖRGYI model of hyperons is discussed.

The notion of the three-dimensional isotopic spin space is useful in a review of the symmetry properties of the strong interactions involving $\pi$- and $K$-mesons. Investigations of GELL-MANN showed that $\pi$-interactions are probably stronger by one order than $K$-interactions, so that he gave them the attributes “very strong” and “moderately strong” [1]. This assumption seems to be supported by experimental work meanwhile performed. Thus it is worth while to seek new symmetry properties of $\pi$-mesons which the $K$-interactions do not possess. (Strong interactions do not allow the application of perturbation calculations, we must therefore welcome even the slightest idea, which helps us to decide, whether an interaction law is right or not, without using the perturbation calculation.)

In his paper quoted above GELL-MANN has himself proposed such a symmetry principle, which is only effective for $\pi$-meson interactions. This is the hypothesis of the global baryon—$\pi$-meson interaction. In formulating the principle, it is important to note, that the $\pi$-interaction of the hyperons of first order ($\Lambda$, $\Sigma^+$, $\Sigma^0$, $\Sigma^-$) shows three-dimensional isotopic invariance both if the four particles are divided into two doublets and if they form a singulet—a triplet. This interesting fact has been emphasized also by SCHWINGER [2], who at the same time proposed a possible explanation in the frame of the four-dimensional isotopic space. He assumed $\pi$, $K$- and electromagnetic interactions to possess four-, three- and two-dimensional isotopic invariance. Usually we go over from the three-dimensional to the two-dimensional isotopic space (two-dimensional subspace of gauge transformations) by considering the 3rd isotopic axis to be distinguished as against the axes 1 and 2. (Electromagnetic interaction is only invariant with respect to the subgroup with one parameter selected out of the three-parameter symmetry group of the
strong interactions.) Unfortunately, in the theory proposed by Schwinger there does not exist a similar visual “geometrical” connection between the four-dimensional (6-parametric) \( \pi \)-symmetry and the three-dimensional (3-parametric) \( K \)-symmetry: in the case of the baryons of even order \( (p, n, \Sigma^+, \Sigma^-) \) another subgroup of the six-parametric symmetry group must be identified with the three-dimensional \( K \)-symmetry group, as in the case of baryons of odd order \( (A^+, \Sigma^-, \Sigma^0, \Sigma^-) \). So we cannot speak of a given four-dimensional isotopic space, in which three- and two-dimensional isospaces are simply subspace.

Now we want to suggest a variant of the theory operating in four-dimensional isospace, which is free from the difficulties mentioned above and is suited, in the first place, to give a review of the relations found by Gell-Mann, Schwinger and others. Finally, we make some remarks on the limits of usefulness of the four-dimensional theory.

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The representations of the Lorentz transformations of the Minkowski space may be deduced from the transformations of spinors with one index. Let us consider the spinor \( a_\mu (\mu = 1, 2) \). The continuous Lorentz group will be represented in the corresponding two-dimensional space by unimodular transformations:

\[
\begin{pmatrix} a_1' \\ a_2' \end{pmatrix} = \begin{pmatrix} a \beta \\ \gamma \delta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad a\delta - \beta\gamma = 1. \tag{1}
\]

An isomorphic representation yields the spinors \( b_\mu \):

\[
\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a \beta \\ -\gamma \delta \end{pmatrix} \begin{pmatrix} b_1' \\ b_2' \end{pmatrix}. \tag{2}
\]

The complex conjugate of \( a_\mu \) will, for example, be transformed according to (2). In the general case \( C_{\nu_1 \ldots \nu_2 j_1 \ldots j_2} \) will be transformed as \( a_{\nu_1} \ldots a_{\nu_2j} b^{i_1} \ldots b^{i_2} \), and yields the representation \( D(jj') \). By complex conjugation we obtain a quantity corresponding to \( D(jj') \). If \( c_{\mu j} \) obeys the reality conditions

\[
c_{11}, \quad c_{22} = \text{real}, \quad c_{12} = c_{21}^* \]

it can be expressed in terms of a usual four-vector, corresponding to \( D \left( \frac{1}{2}, \frac{1}{2} \right) \):

\[
c_{11} = V_0 + V_3, \quad c_{22} = V_0 - V_3, \quad c_{12} = c_{21}^* = V_1 + iV_2. \tag{3}
\]