Some Mathematical Considerations
on Two-mode Searching II
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Along the lines of a previous paper (Seno [2]), making use of an intuitive model, we consider mathematically the relation between the efficiency of searcher's two-mode searching behavior and the target's patchy distribution, and discuss the strategic adaptability of two-mode searching. In this paper, the model is constructed by a time-discrete stochastic process on $S^1$, that is, on a circle. It can be regarded as a modification of Model 2 analyzed in Seno [2]. Differently from Model 2, the searcher's present location is assumed to be influenced by the past passage configuration. This modification yields some particular results for the present model.

Also in the present model, if the patch size becomes sufficiently small, a two-mode searching behavior is strategically adaptable for the searcher. In this model, two-mode searching behavior has high strategic adaptability. Moreover, two-mode searching with an outstanding behavior change is strategically rather adaptable. As for the target's distribution, it appears that a particular patchy distribution is likely to be adopted, depending on the searcher's searching strategy. This result obviously indicates that the target's distribution may be adopted as its evolutionary strategy against the searcher, like a relation between a patchy distributed prey and its predator.

Key words: searching, mathematical model, Markov chain

1. Introduction

In the previous paper (Seno [2]), making use of two simple mathematical models, we considered a coevolutionary game between the searcher's searching behavior and the target's patchy distribution, and demonstrated a strategic adaptability of two-mode searching (i.e., area-concentrated search) depending on the target's distribution strategy. In this paper, we shall consider a coevolutionary game again with an intuitive model constructed by a time-discrete stochastic process on $S^1$, that is, on a circle. The model can be regarded as a modification of Model 2 of the quoted paper. The searcher's present location is assumed to be influenced by the past passage configuration, which is an essentially different assumption from that for Model 2 in the previous paper. We shall see that this assumption carries such a mathematical complexity that the model is not easily analytically tractable anymore. We shall apply the Monte Carlo method to obtain some numerical results, and derive some significant figures to illustrate our argument on the coevolutionary game. The results for the present model show some particular features different from those of Model 2.
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2. Model and Analysis

The model is considered on $S^1$, that is, on a circle (Fig. 1). The modelling space $S^1$ can be regarded as a mathematical translation of the space $\mathbb{R}^1$ where patches are all identical and regularly distributed. The searcher is assumed not to be able to distinguish a visited patch from an unvisited one. In each patch, the targets are assumed to be regularly distributed. Moreover, as the found target is not removed, it is assumed that the searcher cannot distinguish the found target from the encountered one.

![Diagram of model](image)

Fig. 1. Scheme of model. The patch-searching process is terminated when the searcher enters the region $I$ on $S_1$. The target-catching process is subjected to a fixed-giving up step number strategy with $n_c$. For a more detailed explanation, see the text.

For a fixed number of targets within a patch, the higher density of targets will be assumed to imply the smaller patch size. The efficiency of searching a patch and that of catching targets in a patch are not independent.

Our model consists of two processes on $S^1$: i) patch-searching process; ii) target-catching process. In each of these processes, the searcher is viewed as a point moving discretely with a stochastic process. Following a discrete time scale, the searcher discretely changes its site on $S^1$ by each step. The distance between a site and the following site is assumed to be an exponential random variable. The direction of each step is selected at random, that is, with the probability $1/2$ the searcher jumps to the next site in the clockwise or in the anticlockwise direction. The switching rule between two processes is as follows (Fig. 1): The patch-searching process is terminated when the searcher enters a region on $S^1$. This is the moment when the