Cognitive Conflicts in Mathematics Learning

Hartwig Meissner
Westf. Wilhelms-Universität Münster, West Germany

The paper describes research results on how students' conceptions conflict and change in the process of learning mathematics. There are conflicts within the learner which can be explained as developmental conflicts. Some of them seem to be age-dependent (U-shaped behavioral growth: a global view gets destroyed into a chaos and grows up again as a structured view), and others seem to be fundamental as gaps between two types of concepts (spontaneous versus scientific in the sense of Vygotsky). A third kind of conflict appears when the learner expands the boundaries or exceeds the limits of a concept which was successful till then in the limited version. Conflicts between the learner's concept and an outside concept originate from communication problems. Both the learner and the teacher may have correct concepts but there is a mismatch of understanding because of thinking in different frames.

Concepts conflict and change

In general there is no conflict within the learner. The learner has his own consistent «frames», «microworlds», or «domains of experiences» in which he solves a problem according to his cognitive structure and appropriate rules. If there arises a conflict it first is a conflict between the learner and his environment (with the problem, the observer, the teacher, ...). The learner gets confronted with it — consciously or unconsciously — because his actions or reactions obviously do not match the expectations of the environment, because he does not feel the expected agreement from the teacher, because he gets hints for a different view of the problem, or because he cannot solve the problem at all.

If there is no appropriate «correspondance» (Greeno, 1978) between the external message or problem and the internal representation and the general knowledge of the learner he will seek «equilibrium». His reactions may be explained with the Piagetian terms of assimilation and accommodation.

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There is the possibility that the learner maintains his view. A fixed mental image or idea, or mind sets like a visual perception or a «Einstellung» or a functional fixedness prevent the learner from changing his concept. He unconsciously ignores a conceptual conflict and tries to explain his behavior on a side track («it depends how you measure», «the answers in mathematics are not always the same as in real life», «the computer does not work this way», «you should have said that...»), etc.). To resist against a change of his cognitive concept, to persist on the once chosen frame, the learner sometimes even escapes from the original problem and just tries to find out: «What does the teacher want me to say?» As a result of this behavior there will be no cognitive change at all of the learner's concept.

But if the conflict becomes conscious to the learner he mainly tries to adapt or to change his concept. This is not quite easy. The once chosen frame is dominant because it won a competition with others, a competition between several «subjective domains of experiences» (Bauersfeld, 1983). We will discuss these mechanisms in more detail.

**Conflict situations**

There are different kinds of cognitive conflicts. It can be helpful for teachers and educators to know situations in which these conflicts occur. This knowledge then can help to analyse the learner's concepts or misconceptions and it can help to make a conflict conscious to the learner. But the following list will not be a classification of conflicts. More or less we only describe different aspects of the process of learning.

**U-shaped behavioral growth**

We will begin with a description of a developmental phenomenon called U-shaped behavioral growth: «certain behaviors appear, then disappear, only to reappear at a later age» (Strauss, 1979). U-shaped behavioral growth mainly is observed at younger children. These children have a global non-differentiated concept of a certain domain which is appropriate to solve operations or tasks adequately within that domain. The concept is biological in origin and refers to a «common sense knowledge» (Strauss). It is a spontaneous concept in the sense of Vygotsky, based on an intuitive thinking (in the sense of Bruner). But then schooling starts and another concept develops — a «cultural» (Strauss) or «scientific» (Vygotsky) concept — which is reflective and self-conscious, and which is based on analytic thinking.

These two concepts interfere. Abilities relating to the common sense knowledge decrease into a «chaos» while adequate abilities of a cultural knowledge have not yet developed. The global view gets destroyed and the children suddenly cannot solve problems which they could solve before. But step by step the schooling builds up a new and more structured concept. Some of the former abilities «reappear» more powerful than before, now based on a different view. Other abilities are lost for ever. We will give some examples.

In one of our experiments with kindergardeners we presented two sets of little plastic animals, set A with \( n_1 \) blue bears, set B with \( n_2 \) red bears (\( 15 < n_1, n_2 < 20 \) and \( |n_1 - n_2| < 3 \)). The children had to view the two sets for 1-2 seconds and then to decide «is more?» (Lange & Meissner, 1983). See also the experiments from A. Binet (Pollak & Brenner, 1969, p. 86). In our experiments the children of age 4 did significantly better than children of age 6 independently how the elements of the sets were arranged (cf. conservation experiments). The younger children had a better global view of quantity than the older ones who however did better when they got time enough to count the elements of the sets.

A similar observation can be made when playing the game «memory». Young children do much better in remembering partner cards than older children or adults.