Determination of geocenter variations

WU Bin, PEN Bibo and XU Houze (HSU Houze)

Institute of Geodesy and Geophysics, Chinese Academy of Sciences, Wuhan 430077, China

Abstract  The theory and methods to determine the geocenter variations by space geodetic techniques, especially by the satellite laser ranging have been discussed. Then the time series of geocenter variations have been derived from 11-year satellite laser ranging data to Lageos by the adopted methods. The results show that the standard deviations of monthly average values of geocenter in the X and Y directions are 0.2—0.3 cm, and 1.0 cm in the Z direction. The root mean square of residual of the time series of $\Delta X, \Delta Y$ reaches about 0.7 cm. The amplitude of determined ‘peak to peak’ geocenter variation is about 5 cm.

Keywords: geocenter, terrestrial reference frame, satellite laser ranging.

1 Theory

A terrestrial reference frame (TRF), basic frame for the studies of geodynamics, is realized and maintained by a set of coordinates of the global distributed sites which can be determined by Satellite Laser Ranging (SLR), Very Long Baseline Interferometry (VLBI) and Global Positioning System (GPS) techniques. International Earth Rotation Service (IERS) combined the solutions of these different space geodetic techniques and obtained a series of International Terrestrial Reference Frame (ITRF)\(^{11}\). Every ITRF comprised a set of site coordinate vectors and velocities at epoch \(t_0\). The origin of the ITRF was defined as the mass center of the whole Earth including atmosphere, ocean and ground water, while the geocenter was defined as the origin of an Earth crust-fixed coordinate system. As a dynamical conservative system of the whole Earth, mass center remains static in inertial space. Any mass redistribution in the Earth will result in the geocenter variation relative to the mass center. Therefore, for a terrestrial reference frame, mass center and geocenter coincided at time \(t_0\), but unnecessarily coincided after \(t > t_0\). At present, of the three space techniques used to realize the ITRF, VLBI can be used to realize the crust-connected geometric reference frame, while its origin is irrelative to the mass center. SLR and GPS both track artificial satellites and are used to realize the TRF based on the dynamical satellite orbit theory. As the satellite orbit is sensitive to the geopotential variation and the first degree spherical harmonic coefficients represent the geocenter variation, SLR or GPS realize TRF with its origin relative to the mass center. However, instead of by direct ranging measurement, the high precise GPS observable is the double-differenced phase which decreases the sensitivity to geocenter variation. There leaves the SLR as the only space technique to both realize TRF and mass center as its origin with centimeter precision.

In a TRF, the \(j\) site coordinate vector at time \(t\) is written as

$$R_j(t) = R_j(t_0) + V_j \times (t - t_0) + \Delta R_j(t),$$  \hspace{1cm} (1)

where \(R_j(t_0)\) is site \(j\) vector at reference epoch \(t_0\), \(V_j\) is the velocity vector, and \(\Delta R_j(t)\) is periodic variation of coordinate vector. Recent studies have manifested that the accuracy of the determined \(R_j(t_0)\) by space techniques is better than one centimeter, and the determined \(V_j\) agreed with the geological model in a few millimeter per year\(^{2}\). Eq. (1) has two meanings. The first one is that one can calculate the \(R_j(t)\) by the equation at time \(t\). The second is that one can determine the \(R_j(t_0)\) and \(V_j\) by using the measurements around \(t_0\). Because the SLR realizes TRF at reference epoch \(t_0\) and ties its origin to the mass center, for an observer on the Earth crust, the origin of the TRF at \(t\) is different from the one at \(t_0\) that makes eq. (1) modified in order to take account of the effect of geocenter variation on \(R_j(t)\) in TRF. Let the geocenter variation relative to the mass center be \(\Delta R_c(t) = (\Delta X, \Delta Y, \Delta Z)\), the site coordinate vector in TRF realized by SLR at \(t\) is written as

$$R_j(t) = R_j(t_0) + V_j \times (t - t_0) + \Delta R_j(t) + \Delta R_c(t),$$  \hspace{1cm} (2)

The difference between eq. (1) and (2) is that if we want to calculate the site coordinate vector at time \(t\) in TRF relative to geocenter, eq. (1) will be still valid, but if we want to calculate the vector relative
to the mass center, eq. (2) should be used instead of eq. (1). Another importance of eq. (2) lies in the fact that the secular change of geocenter may result in a few millimeters effect on SLR determined $V_j$.

In addition, the geocenter variation can also be represented by the first degree spherical harmonic geopotential coefficients $C_{11}$, $S_{11}$ and $C_{10}$ as the following equation:

$$\Delta R_o(t) = -R_e \times (C_{11}, S_{11}, C_{10}),$$

(3)

Where $R_e$ is the equatorial radius of the Earth. Any centimeter level geocenter variation will cause $C_{11}$, $S_{11}$ and $C_{10}$ with amplitude $10^{-9}$, so we can use either eq. (2) or (3) to determine the geocenter variation $\Delta R_o(t)$.

2 Results and discussion

As discussed above, according to eqs. (2) and (3) by analyzing the SLR data to Lageos from September 1983 to December 1994, we obtain the geocenter variation $\Delta R_o(t)$. Our analysis shows that the result using eq. (2) or (3) is almost the same. In this analysis, the models and constants are adopted according to IERS convention$^{[1]}$ in our SLR software IGGSRL. It is convenient to divide the 11-year data into 136 monthly sub-arcs and the determined parameters into common parameters and local parameters. The common parameters include $R_o(t_k)$ and $V_j$, which are estimated once in 11 years, while the local parameters comprise geocenter variation $\Delta R_o(t)$, satellite initial position vector $r(t_k)$ and velocity vector $v(t_k)$, solar pressure coefficient $C_s$, empirical drag coefficient and its rate $D_s$, estimated monthly in every sub-arc. The Earth's rotation parameters and the initial site coordinates are taken from the IERS reports and ITRF94 respectively.

Figures 1—3 show the monthly average values of $\Delta X$, $\Delta Y$ and $\Delta Z$ from September 1983 to December 1994. The standard deviation (SD) ranges from 0.2 to 0.3 cm for $\Delta X$ and $\Delta Y$, but 1.0 cm for $\Delta Z$, which means that the precision of $\Delta Z$ is much less than $\Delta X$ and $\Delta Y$. The possible reason is that Lageos orbits is in the polar plane (inclination = 109°) and the tracking stations are mainly located in the North Hemisphere. So the strength of observational geometry is not high enough for $\Delta Z$ precise determination. In further work we will analyze both Lageos and Lageos-2 (inclination = 52°) data to strengthen the observational geometry.

In the following, we only discuss the $\Delta X$ and $\Delta Y$ components. In general, there are two indexes to evaluate our results. One is the SD of the monthly determined $\Delta X$ and $\Delta Y$ which represents the limit precision for the $\Delta X$ and $\Delta Y$ determination by using present SLR data. The other is repeatability of the time series of $\Delta X$ and $\Delta Y$. The recent theoretical calculations$^{[3]}$ suggest that the amplitude of geocenter variation neither exceeds 1 cm nor changes strongly month by month. Hence the root-mean square of residuals (RMS) of $\Delta X$ or $\Delta Y$ time series is used as an important index to evaluate the accuracy of the geocenter variation determination. From figs. 1 and 2, the RMS for $\Delta X$ and $\Delta Y$ are 0.73 and 0.71 cm respectively. The ‘peak to peak’ values of $\Delta X$ and $\Delta Y$ are about 5.0 cm.

Fig. 1. Geocenter variation $\Delta X$. 