DETERMINATION OF PRINCIPAL STRESSES IN AN ISOTROPIC MATERIAL UNDER CONDITIONS OF PLANE STRAIN

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Summary

Starting from basic considerations, equations have been developed for the determination of principal stresses in an isotropic material, under the action of a twodimensional stress system. It has been shown that the slip lines, in the case of a plastic state of stress; and the shear stress pattern and the isoclinics in the case of an elastic state, together with the boundary conditions, provide enough data for solving any specific problems. An example has been given to show that the equations developed hold for the perfectly plastic state. But the authors are precluded from giving a numerical example for the general plastic condition, as the required experimental data are not available to them. In the case of a loading producing a completely elastic state of stress throughout the material a numerical example has been worked out, based on published experimental data, showing the actual application of the proposed equations. The method is, perhaps, not applicable in a case where plastic and elastic states of stress coexist.

Nomenclature.

$\sigma$, $\tau$ Cartesian coordinates.

$r$, $\theta$ polar coordinates.

$f_1$, $f_2$ principal stresses.

$t_x$, $t_y$ normal stresses.

$\tau_{xy}$ shear stress.

$p$ average stress.

$K = f_1 - f_2$.

$\sigma$ stress function.

$\alpha$ slope of a characteristic.

$\beta$ slope of the larger principal stress.

Other symbols are defined in the text where they occur for the first time.

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§ 1. Introduction. We shall consider a two-dimensional state of stress in the isotropic material and restate some of the well-known relations \(^1\) in our notation, in order that the subsequent sections become clearer.

The equilibrium conditions can be written as

\[
\frac{\partial f_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0; \quad \frac{\partial f_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0.
\] (1.1)

If the stress function is \(\varphi\), then (1.1) will be satisfied when

\[
\varphi_{xx} = f_y, \quad \varphi_{yy} = f_x \quad \text{and} \quad \varphi_{xy} = -\tau_{xy},
\] (1.2)

where \(\varphi_{xx}, \varphi_{yy}, \varphi_{xy}\) are the second order partial differential coefficients of \(\varphi\).

The principal stresses are related to the normal and shear stresses by the expression \(^3\)

\[
(f_1 - f_2)^2 = (f_x - f_y)^2 + 4\tau_{xy}^2,
\] (1.3)

where we can put

\[
f_1 - f_2 = K,
\] (1.4)

such that \(K\) is a function of the coordinates \((x, y)\).

Equation (1.3) is the differential equation for the stresses. It is non-linear, partial and of the second order. It can be solved by the usual methods of characteristics \(^3\). The discriminant of the equation is positive, hence the characteristics are real. Their directions are given by

\[
\tan 2\alpha = \frac{(f_x - f_y)}{(2\tau_{xy})},
\] (1.5)

where \(\alpha = \arctan \left(\frac{dy}{dx}\right)\). On the other hand, if \(\beta\) is the angle made by the direction of the larger principal stress with the x-axis, we can write, assuming that the average stress is \(\rho = \frac{1}{2}(f_1 + f_2)\),

\[
\begin{aligned}
f_x &= \rho + \frac{1}{2}K \cos 2\beta, \\
f_y &= \rho - \frac{1}{2}K \cos 2\beta, \\
\tau_{xy} &= \frac{1}{2}K \sin 2\beta.
\end{aligned}
\] (1.6)

Hence, it is seen that

\[
\tan 2\beta = 2\tau_{xy}/(f_x - f_y).
\] (1.7)

Combining equations (1.5) and (1.7) we obtain

\[
\tan 2\alpha \tan 2\beta = -1.
\] (1.8)