MECHANICS OF ADHESIVE JOINTS

PART I. RESIDUAL STRESSES

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Summary

In order to understand precisely the nature of adhesive strength, we must at least have exact knowledge of the stress distribution in the adhesive layer arising from residual strain as well as from loading.

In this paper we deal with the thermal stress as a representative residual stress, which occurs during gluing operations. Reducing the problem of the thermal stress to one of boundary traction, the stress function of a rectangular plate whose two parallel opposite edges have been glued to two rigid planes respectively is solved.

The results obtained are found to be in good agreement with those of photoelastic experiments even in the case of a thin plate whose relative dimension is comparable to that of actual adhesive joints. In general the stress distribution changes sharply in the vicinity of the peripheral edge of the cemented plane where some stress concentration occurs. Such a disorderliness of stress distribution, however, diminishes within a short distance from the free edge, and beyond this region the stress distribution in the adhesive layer becomes uniform just as in the case of uniaxial tension.

§ 1. Introduction. Of the significant problems concerning the stability of adhesive joints that of the injurious effects of residual stresses 1), such as those resulting from the contraction of an adhesive layer in bulk, is most important. Besides the direct effect of shrinkage caused by curing an adhesive, evaporating solvent or desorbing moisture, the difference of thermal strain of adhesive and adherent, which happens whenever the temperature changes, also causes such internal stresses. The intensity of stress depends, of course, on the magnitude of the strain and the modulus of elasticity, and sometimes it may become high enough to cause spontaneous destruction of the joint.
In this paper, as a typical case of residual stress, the thermal stress is dealt with, in view of the expectation that the stress distribution of other cases can also be derived from the same stress function so long as the adhesive layer is homogeneous in nature. As the length and the width of the adhesive layer are both sufficiently large compared with the thickness, the problem of thermal stresses may well be treated in terms of plane strain.

B. J. Aleck 2) has already obtained an approximate solution of the thermal stress of a rectangular plate, whose one edge is cemented on a perfectly rigid body (while the other three edges are free) by the method of minimum strain energy. His results are, however, rather curious: abnormally high values of normal and shearing stresses are found at the corner of the cemented edge, amounting to about ten times and three times the average interfacial thermal stress respectively. Moreover, the stress distribution obtained seems to coincide scarcely with that shown by the photoelastic experiments. Such illusory results may be ascribed to the too simplified stress functions which he took as basis.

Accordingly, we have attempted to solve strictly the similar problem of a rectangular plate, whose two parallel opposite edges are cemented to two rigid plates respectively, using as the stress function the double harmonic functions and replacing the thermal problem by that of a boundary traction. The results are quite satisfactory; we have found the stress distribution to be perfectly in line with that shown by the photoelastic experiments and the solution to hold exactly even in a thin layer as is the case for actual adhesive layers.

§ 2. The treatment of the thermal stress as a boundary traction problem. We have first replaced the thermal stress problem by that of a specified surface traction, using the procedure proposed by B. J. Aleck 2). Consider a rectangular plate of constant thickness whose height is $2h$ and whose width is $2l$. Two opposite sides of the edges of the plate of length $2l$ are cemented to two adherent planes respectively at a temperature $T$, leaving the other two edges free. The plate material is assumed to be elastic and isotropic. The sign convention and directions of the coordinate system are indicated in fig. 1. The problem is to determine the stresses produced by the decrease of temperature from $T$ to $T - t$. If it is assumed that the