THE CONTRIBUTION OF RADIATION TO THE
CONDUCTION OF HEAT

II. BOUNDARY CONDITIONS *)

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Summary
In part I the boundary conditions were left out of consideration. In this part we shall deal with the boundary conditions in the stationary cases, the non-stationary problems being as yet too complicated to be solved generally.

If the surfaces of a body are sufficiently far apart, the phenomena at the surfaces can be described by a transmission coefficient \( \alpha_0 \), which in the dimensionless form \( \alpha_0/\lambda_{\infty} \) is only a function of properties of the materials used (emissivity of the bounding materials, ratio of absorption and scattering in the body and ratio of pure and radiant conductivity). Its influence can only be lowered by changing these properties, but cannot be completely suppressed.

In thin layers (optical thickness smaller than \( 10/\epsilon p \)) the influence of both surfaces has to be taken into consideration. The integro-differential equations controlling the phenomena and approximate solutions are given. Theory and experiments are in agreement. As an example kapok (weight \( \epsilon \) > 0.0017 g/cm\(^3\)) is investigated: \( \epsilon = 794 \) g; \( \varphi = 0.20 \) (i.e.: 20% of the extinction coefficient is absorption and 80% is scattering).

§ 6. Steady state for boundary conditions without mutual influence of the surfaces. First we shall consider the phenomena in the neighbourhood of an isothermal metal surface and suppose that the problem can be treated as one-dimensional (the radiation in reality makes it three-dimensional). We assume that \( \beta, \sigma, \epsilon \) and \( \epsilon \), the absorption, the scattering and the extinction coefficients in the matter and the emissivity of the metal surface respectively, are independent of the wave-length and that \( n_{r} = 1 \). We introduce the following symbols: \( \varphi = \beta/\epsilon, \tau = \epsilon x \), the optical distance, \( x \) being the distance

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from the surface, and in a following section $D = ed$, the optical thickness, $d$ being the thickness of a layer.

Other surfaces may be so far away that they cannot disturb the phenomena considered. The temperature of the surface is taken to be the zero point of the temperature scale and we assume the temperature to rise with increasing distance from the boundary. The temperature distribution turns out to be as given in fig. 1. Far from the surface this distribution can be described by a straight line with a slope $H$. Extending this straight line to the surface we get an intersection with the ordinate at $\theta_0$. The heat flux is $q = -\lambda_m H$ ($\lambda_m = \lambda + 4a_\nu/3e$). Varying $q$ makes all values of the temperatures $\theta$ and $H$ vary proportionally under otherwise equal circumstances. Thus $\theta_0$ can be taken as a characteristic temperature difference.

![Fig. 1. The temperature distribution in the neighbourhood of a surface of a material with high extinction coefficients.](image)

We therefore divide (1) in part I by $\theta_0$ and introduce $\theta = \theta/\theta_0$. We can put

$$\frac{4}{\theta_0} \int_{0}^{\infty} \beta I_{1,T} \, dl = \frac{4}{\theta_0} \beta \left( I_{1,T_0} + \frac{\partial I_{1,T}}{\partial T} \theta \right) \, dl = \frac{4}{\theta_0} \beta \int_{0}^{\infty} I_{1,T_0} \, dl + 4\beta a_\nu \theta$$

with the known restriction $|\partial T/\partial n| < 0.004 \varepsilon T$. In a similar way we change $(4/\theta_0) \int_{0}^{\infty} \beta J_1 \, dl$ into $(4\beta/\theta_0) \int_{0}^{\infty} I_{1,T_0} \, dl + 4\beta a_\nu F$ *) in which we suppose $F$ as well as $\theta$ to be only a function of the optical distance $T$. Then (1) in part I is changed into

$$F = \theta - \frac{\lambda e}{4a_\nu} \frac{d^2 \theta}{d\tau^2}$$

*) This $F$ is the $F$ in the first article divided by $[\partial I_{1,T}/\partial T]_0$. It is a dimensionless quantity.