The Johnson-Mehl-Avrami-Kolmogorov model: 
A brief review (*)

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Summary. — The kinetics of isothermal phase transformations have been described by the Johnson-Mehl-Avrami-Kolmogorov (JMAK) phenomenological model since the 40’s. Although many theoretical investigators have given a fundamental contribution to extend the range of applicability of the model, the experimentalists keep using the model in its original form, owing to its simplicity, at times risking to overinterpret the experimental results. The early attempts to apply the JMAK model to surface science date back to the 70’s, but it has been systematically applied only in the last few years. There is not enough space here to review thoroughly the model and its applications. For this reason, after a historic introduction, we describe here in some detail the model and finally we review in brief its application to surface science.

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1. – Introduction

If one types the word Avrami as entry of the facility search of the Internet Elsevier site, one receives more than one hundred titles. Those are works published in the last two years in the entire set of scientific journals printed by Elsevier and dealing with a phenomenological model describing the kinetics of isothermal phase transitions which proceed through nucleation and growth, also known as Johnson-Mehl-Avrami-Kolmogorov (JMAK) model [1-3]. Although this model dates back to the end of the 30’s and the beginning of the 40’s, it has been used and overused until nowadays basically for its extreme simplicity. As a matter of fact, the essence of the model can be summed up through a very simple formula

\[ V(t) = 1 - e^{-V_0 t}, \]

(*) In honour of Prof. Gianfranco Chiarotti on the occasion of his 70th birthday.
where \( V \) is the fraction of the transformed phase and \( V_e \) is the so-called extended volume of the transformed phase, that is the volume the transformed phase would acquire if the overlap among the growing nuclei were disregarded; once the extended volume had been determined, the kinetics would be exactly known. Before going through the derivation of eq. (1) and discussing its values and limits, it is worth retracing in brief when, where and what the four scientists have written about the model named after them.

Although Kolmogorov was a mathematician, indeed one of the greatest mathematicians of our century, it can be asserted, quite confidently, that eq. (1) is the result of a big effort that metallurgists made in order to describe and deeply comprehend isothermal reactions such as the process of freezing or of recrystallization of pure metals and alloys or the eutectoid decomposition of a solid solution phase. The physical metallurgy is having a great moment at the end of the 40’s, quoting R. F. Mehl: "...facts and theories, foregathered on a wide front, and physical metallurgy had truly come of age" [4]. Kolmogorov, in the introduction of his paper, states that his work arises to solve a metallurgical problem that his friend I. L. Mirkin proposed to him. Kolmogorov solves the problem in a quite general way by using the theory of probability and applies the model to the case of constant and simultaneous nucleation. This work, written in Russian and published in Soviet Union in 1937, remains unknown in the United States and western Europe, and two years later Johnson and Mehl achieve the same results but through a more involved mathematical pathway. It is worth reporting a sentence of Evans, who, in 1945, published the demonstration of the JMAK formula by using the theory of probability. He writes: "It is difficult to believe that the method has not been published before; but, if it is not new, the papers describing it are unknown to the author and to those whom he has consulted on the matter..." [5]. Between 1939 and 1940 Avrami publishes two, now famous, works on the subject. In the first the author gives a formal solution of the kinetics of growth in terms of a series. The demonstration is rather elegant and basically founded on the set theory. Incidentally Avrami’s series is nothing but the formula for estimating the probability of occurrence of an event \( A \) composed of an infinite number of not mutually exclusive events \( A_k \) [6]. On the other hand, the axiomatic theory of probability due to Kolmogorov, based on the modern metric theory of functions and on set theory, was born just in that period. Unfortunately the series is not manageable due to its complexity, and so useless for getting meaningful information from real kinetics. Aware of that, Avrami declares that, in a subsequent paper, he will give an approximate solution by developing the first two terms of his own series. This is an interesting fact from the historical point of view. Actually he will never keep his word and, in the second work (1940) [2], he goes beyond his promises and publishes the right general solution. Besides, he introduces the concept of phantom grains (see below), that is those grains which start growing after they had been swallowed by one or more nuclei. Although they do not contribute manifestly to the transformed fraction of the new phase, yet, as Avrami for the first time puts in evidence, they are fundamental in order to obtain the correct description of the kinetics. Nevertheless, by reading carefully the discussion that follows the contribution of Johnson and Mehl (1939) [3], it clearly appears that, at that time, Avrami still had not understood the role of phantoms. He will mature this concept and its importance, and with it he will find the correct solution, only one year later.