The Role of Task Format, Mathematics Knowledge, and Creative Thinking on the Arithmetic Problem Posing of Prospective Elementary School Teachers

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A Test of Arithmetic Problem Posing was developed by the authors to examine the arithmetic problem-posing behaviours of sixty-three prospective elementary school teachers. Results of analysis were then used to examine task format (i.e., the presence or absence of specific numerical information) on subjects' problem posing and the relationship between subjects' problem posing and their mathematics knowledge and verbal creativity. The major findings were that the test effectively evaluated arithmetic problem posing, and that most subjects were able to pose solvable and complex problems. In addition, problem-posing performance was better when the task contained specific numerical information than when it did not, and that problem-posing performance was significantly related to mathematical knowledge but not to verbal creativity.

Introduction

Recent professional reports advocating mathematics education reform have called for an increase in problem-posing activities such as emphasizing student-generated problems, in addition to having students solve problems which have been posed for them by the teacher or the textbook (NCTM, 1989; NCTM, 1991). In fact, because of its centrality to the doing of mathematics, problem posing has been identified by some mathematicians (see, for example, Freudenthal, 1973; Polya, 1945; Silver, 1994) as a critically important aspect of mathematics education. With the inclusion of problem posing in classrooms, a wide range of problem-posing activities can be undertaken, but it is the more constrained types of problem posing that are likely to receive greater initial attention from classroom teachers. Some previous research (e.g., Keil, 1965; Perez, 1985; Winograd, 1991) has shown the instructional efficacy of such simple instructional activities as having students generate or rewrite mathematics problems as a means of improving their problem solving and their attitude toward mathematics. Unfortunately, in such research, the quality of the problems that students generated has not been measured. Thus, the development of techniques for evaluating posed problems is needed to support research aimed at examining the quality of such problems.

Goals and Conceptual Framework for This Study

A major goal of this study was to develop and apply a cognitive analysis scheme to examine the problem posing of a group of prospective elementary school teachers. The decision to use teachers as subjects in this study was based on a belief that it was important to understand their problem posing not only because they represented a population of mature, yet novice, posers of mathematics
problems, but also because an understanding of their problem posing could help inform efforts to promote a greater emphasis on problem posing in mathematics instruction. In order to examine the problem posing of prospective elementary school teachers, a test of arithmetic problem posing, which included a cognitive analysis scheme, was developed and administered.

The form of problem posing studied in this investigation is arithmetic problem posing, in which one or more arithmetic story problems is generated from a situation presented in story form. In this case, the products of mathematical problem-posing activity are themselves problems. Therefore, it was possible to adapt techniques and constructs available from the extensive research on mathematical problem solving, and especially arithmetic problem solving, to analyse the products of such problem-posing activity.

A second goal in this study was to examine the influence of two different task formats on problem posing. Task variables have been extensively studied in the area of mathematical problem solving (Goldin & McClintock, 1979). In particular, research has shown that several different format variables, such as the number and order of words or the presence/absence of figural information, can influence arithmetic problem solving performance under some conditions. In this study, a task format variable was defined as the presence or absence of specific numerical information. In a prior study on solving, it has been found that subjects' solving ability can be negatively influenced by the presence of numbers which are irrelevant to a problem's solution (Reusser, 1986). One might then predict that subjects' problem posing would be enhanced by the absence of specific numbers, which would allow subjects to focus on deeper structural aspects of the problems to be posed. On the other hand, one might also predict that the absence of specific numbers might hinder novice subjects' problem generation. By requiring subjects in the study reported here to respond both to a task containing specific numerical information and to a task not containing specific numerical information, the effect of this task format variable on problem posing was examined.

A third goal in this study was to examine problem-posing in the light of differences among prospective teachers' general mathematics knowledge and creativity. The reason for studying mathematical background knowledge is fairly straightforward. It can be argued that, in order to generate a reasonable mathematical problem related to a given situation, one must (a) be aware of facts and relations embedded in the situation; (b) be able to mathematise the situation; and (c) be able to present one's mathematised situation in the form of a problem. In fact, some previous research has pointed to differences in problem posing between subjects with high mathematics ability and those with low mathematics ability. For example, Ellerton (1986) reported that more capable students appeared to be more thoughtful than less capable students in their problem posing, as evidenced by their apparent planning in generating problems (for example, if they included fractions in a multiplication problem, then they tended to use fractions which allowed for cancellation in order to simplify the computation). In other research, Krutetskii (1976) gave students problem stems without a question and asked them to pose a problem. He reported that high ability students could "see" those problems that "naturally followed" from the given information, whereas low ability students did not do so, even when hints were given by the interviewer.