Transfer functions attached to linear systems with time varying parameters

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Abstract

To obtain transfer functions attached to linear time varying (LTV) systems, a new method for getting the poles and residia of linear time invariant (LTI) continuous or discrete systems is proposed. The explored method is superior to others known, because it can be extended to systems with time varying parameters. With the poles and residia so obtained, the transfer function attached to the LTV systems, both continuous and discrete, results easily.

Keywords: Transfer function, Linear system, Invarying system, Time variable system, Continuous system, Discrete system, Pole, Residue.

FONCTIONS DE TRANSFERT DES SYSTÈMES LINÉAIRES À PARAMÈTRES VARIABLES DANS LE TEMPS

Résumé

Pour obtenir les fonctions de transfert des systèmes linéaires variables dans le temps, une nouvelle méthode pour obtenir les pôles et les résidus des systèmes linéaires inva-
riables, continus ou discrets, est proposée. La méthode explorée est supérieure à d'autres connues parce qu'elle peut être prolongée aux systèmes à paramètres variables dans le temps. Avec les pôles et les résidus ainsi obtenus, la fonction de transfert des systèmes linéaires variables, continus ou discrets, peut être facilement déterminée.

Mots clés : Fonction transfert, Système linéaire, Système invariant, Système variable temps, Système continu, Système discret, Pôle, Résidu.

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A linear time varying system has parameters that vary with time and are often modeled by linear differential or difference equation, which have time varying coefficients [1, 2]. Linear time varying filters and automatic control systems driving a LTV process [3] belong to this category. In both cases, no classical method can be used to get their transfer function. Problems concerning time varying systems as transfer functions attached to them are studied in references as [4, 5, 6, 7]. For discrete systems, the generalized transfer function (GTF) can be obtain by frozen time approximation [8], but unfortunately, it is available only at a frozen time instant. In [9] a new method to estimate reliable time-varying transfer functions and time variant impulse response was introduced. The method is based on time variant autoregressive moving average models that is more accurate than the recursive least-squares.

The present explored method allows us to determine the poles and residia of the LTV systems in a simple manner. To perform this, a new method for determining the poles and residia of a discrete time invariant system is provided in Section II. In Section III the proposed method is extended to a discrete linear time varying system. A similar new method to get the poles and residia of a continuous time invariant system is presented in Section IV and in Section V the method is extended to continuous linear time varying systems. Section VI presents a simulation case study that reveals a pictorial view of the pole location of the considered system with respect to the unit circle. Section VII concludes the paper.

II. CALCULUS OF THE POLES AND RESIDIA OF DISCRETE SYSTEMS WITH TIME INVARIANT PARAMETERS

Let us consider a discrete linear time invariant system described by a difference equation of first order:

\[ y[n] - a_1 y[n - 1] = x[n] \]  

Introducing the delay operator \( D \), that is:

\[ y[n - k] = D^k y[n] \]

(1) may be written as:

\[ y[n] = \frac{x[n]}{1 - a_1 D} \]

Assuming zero initial conditions, the output can be computed as follows:

\[ y[n] = \sum_{k=0}^{n} x[k] a_1^{n-k} \]

From (3) and (4), we have:

\[ \frac{x[n]}{1 - a_1 D} = \sum_{k=0}^{n} x[k] a_1^{n-k} \]