A KIND OF NONLINEAR OSCILLATIONS OF SINGLE DEGREE OF NON-AUTONOMY SYSTEM*

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Abstract

In this paper we use the method of derivative expansion of multiple scales of singular perturbations, and we have solve the forced vibration equation of a particle attached to a nonlinear spring under the influence of slight viscous damping. The problem is of the fourth order nonlinearity. The four cases discussed are: the soft excitation of non-resonance, the hard excitation of non-resonance, the soft excitation of resonance, the hard excitation of resonance.

I. Introduction

The research into the problem of nonlinear system usually becomes of nonlinear differential equations. Mathematically this may fall into difficulty. At present, only a few special kinds of nonlinear equations can have accurate analytic solutions, but a large number of nonlinear equations can not.

In production it is necessary to develop approximate calculations. Many approximate calculations can be found in ref. [2] which recommends a lot of reference literature. Most of the calculation methods therein discuss the system with quadratic and cubic nonlinearity and typical cases are oscillation Duffing equation, Vonstpol equation and spring pendulum and so on. But the fourth order nonlinearity has been rarely dealt with, this paper attempts to go a step further than ref. [4], to point out that ref. [4] is mistaken there, and we researched the soft excitation which has not been touched by ref. [4].

II. The Problem

The equation of a kind of nonlinear oscillations of single degree of non-autonomy system is:

\[ \frac{d^2u}{dt^2} + \omega^2 u = -2\epsilon \mu \frac{du}{dt} - \epsilon \alpha u^4 + K \cos \Omega t \]

in which \( \omega \) is the natural frequency, \( \epsilon \) is dimensionless parameter, \( \mu \) is the viscous damping coefficient, \( \alpha \) is constant, \( K \) is amplitude of excitation and \( \Omega \) is frequency of excitation.

Here are many cases. When a force of excitation is smaller, it is called a soft excitation and \( K = O(\epsilon) \). When a force of excitation is greater it is a hard excitation and \( K = O(1) \). When the frequency of excitation is equal to the natural frequency, \( \Omega - \omega = O(\epsilon) \), the resonance namely

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called principal resonance. When one of combined frequencies is close to the natural frequency of system, \( n\Omega + m\omega \approx \omega \). \( m, n \) are integers, resonance also takes place. When \( |\Omega - \omega| \) and \( |(n\Omega + m\omega) - \omega| \) is greater numbers, nonlinear oscillation is of the non-resonant motion.

The four cases we have researched into are as follows:

1. Hard excitation of non-resonance
   \[ K = O(1), \quad \Omega - \omega = O(1) \text{ or } n\Omega + m\omega - \omega = O(1); \]

2. Hard excitation of resonance
   \[ K = O(1), \quad \Omega - \omega = O(\epsilon) \text{ or } n\Omega + m\omega - \omega = O(\epsilon); \]

3. Soft excitation of non-resonance
   \[ K = O(\epsilon), \quad \Omega - \omega = O(1) \text{ or } n\Omega + m\omega - \omega = O(1); \]

4. Soft excitation of resonance
   \[ K = O(\epsilon), \quad \Omega - \omega = O(\epsilon) \text{ or } n\Omega + m\omega - \omega = O(\epsilon). \]

III. Solution of the Problem

1. Hard excitation

   \[ K = O(1) \quad (3.1) \]

   The chosen time scales are:

   \[ T^n = e^{nt} \quad (n = 0, 1, 2, \ldots) \]

   We choose only two time scales here. That are:

   \[ T_0 = t, \quad T_1 = et \quad (3.2a,b) \]

   Assume expansion for \( u \) as:

   \[ u = \sum_{n=0}^{M-1} e^n u_n(T_0, T_1, T_2, \ldots, T_M) + O(e^M) \quad (3.3) \]

   Here \( u \) is taken:

   \[ u = u_0(T_0, T_1) + O(\epsilon) \quad (3.4) \]

   Depending on method of multiple scales:

   \[ \frac{d}{dt} = \frac{\partial}{\partial T_0} + e \frac{\partial}{\partial T_1}, \quad \frac{d^2}{dt^2} = \frac{\partial^2}{\partial T_0^2} + 2e \frac{\partial^2}{\partial T_0 \partial T_1} + e^2 \frac{\partial^2}{\partial T_1^2} \]

   There are:

   \[ \frac{du}{dt} = \frac{\partial u}{\partial T_0} + e \frac{\partial u}{\partial T_1}, \quad \frac{d^2 u}{dt^2} = \frac{\partial^2 u}{\partial T_0^2} + 2e \frac{\partial^2 u}{\partial T_0 \partial T_1} + e^2 \frac{\partial^2 u}{\partial T_1^2} \quad (3.5a,b) \]

   Substituting (3.4) and (3.5a, b.) into (2.1) and equating the coefficients of power of \( \epsilon \) yields:

   \[ \frac{\partial^2 u_0}{\partial T_0^2} + \omega^2 u_0 = K \cos \Omega T_0 \quad (3.6) \]

   \[ \frac{\partial^2 u_1}{\partial T_0^2} + \omega^2 u_1 = -2 \frac{\partial^2 u_0}{\partial T_0 \partial T_1} - 2\mu \frac{\partial u_0}{\partial T_0} - au_0^4 \quad (3.7) \]