Summary - The Bonferroni index (BI) and Bonferroni curve (BC) have assumed relief not only in economics to study income and poverty, but also in other fields like reliability, demography, insurance and medicine. Besides, the increasingly frequent comparison with the Lorenz curve (LC) and Gini index (GI) both in theoretical and applied studies has driven us to derive explicit expressions for BI, BC, GI and LC for some thirty five continuous distributions. It is expected that these expressions could provide a useful reference and encourage further research within the aforementioned fields.

Key Words - Bonferroni curve; Bonferroni index; Continuous distributions; Gini index; Lorenz curve.

1. Introduction

In this paper we consider the Bonferroni index (BI) and the Bonferroni curve (BC) besides the well known and broadly used Gini index (GI) and Lorenz curve (LC) because of some suitable properties of the former in comparison to the latter.

BI and BC have begun to receive the due attention only in recent years, both because they were proposed in a book in Italian (Bonferroni, 1930) and for the aversion of Gini, also underlined in international congresses, for any other inequality index which could overshadow his famous GI also known as concentration ratio (Gini, 1914). In fact, only several years after the death of Corrado Gini, who had influenced the statistical research in Italy (see Giorgi (2001)) and who had a relevant scientific international weight for about forty years, BI and BC were again objects of study. Piesch (1975) and Nygard and Sandström (1981) were the first to investigate the subject in their books on the measurement of (income) inequality. Some decades before De Vergottini
(1950) proposed a general formula for inequality measures of which the Gini and Bonferroni indices are special cases.

The recent contributions mainly concern some aspects like poverty, welfare, decomposability, reliability, sampling and inference.

The first aspect is surely the most remarkable, in fact the greater sensitivity of BI in comparison to GI to lower levels of the income distribution makes it a suitable tool for analyzing poverty. In this context Giorgi and Crescenzi (2001a) have proposed some poverty measures based on BI (see also Chakravarty and Muliere (2004)).

As far as the different sensitivity of BI and GI to the transfers in some parts of the (income) distribution is concerned, it is necessary to point out that few summary measures of inequality, which supplement each other, can be used together to have better information on the distribution shape. The topic has been carefully investigated in two very interesting studies by Aaberge (2000, 2007). In the first paper, starting from the analogy between the LC and the cumulative distribution function, he has proposed to synthesize the LC information through the first few LC-moments. Subsequently, since none of these indices, labelled Lorenz family of inequality measures, is particularly sensitive in the lower part of the distribution, Aaberge (2007) has overcome this drawback through a transformation of the LC, called Scaled Conditional Mean Curve (SCMC). The latter is equivalent to the LC, it is connected to the so-called “decile approach” and, above of all, unlike the LC, it is “strongly related to the shape of the underlying distribution function”. Aaberge has suggested to use jointly the first three order moments of the SCMC because each of them is particularly sensitive, respectively, in lower, central and upper part of the distribution. He also has shown that the set of these three measures, called Gini’s nuclear family, includes both the Bonferroni index and the Gini coefficient. Such results strengthen the possibility to use BI both alone, in the study of poverty, and jointly with other indices, included GI, to supplement them(1).

Welfare implications of BI have been studied by Benedetti (1986), Aaberge (2000) and Chakravarty (2007). The latter has reached very interesting results investigating besides the Bonferroni Social Welfare Function (BSWF) the relationships of BI with GI and the consistency of BI with several types of income distributive principles.

(1) Aaberge’s (2000, 2007) papers, whose most important aspects for the purpose of our work we have synthetically underlined, are a remarkable source from which drawing further results. We refer the reader to these articles in order to go into other aspects, e.g., the explicit expression of some inequality indices in terms of welfare function and the normative foundations, included BI and GI. Asymptotic distribution aspects for the empirical SCMC and the related empirical inequality indices are also analyzed. Finally, it is to be noted that the scaled conditional mean curve, labelled here SCMC synthetically, can also be seen as an alternative way to express the Bonferroni curve.