Extended skew generalized normal distribution

Summary - The skew normal distributions’ family represents an extension of the normal family to which a parameter ($\lambda$) has been added to regulate the skewness of the distribution. In recent years, not only the skewness but the kurtosis is also of more concern in representing the features of the distribution. In this study a more flexible distribution, extended skew generalized normal distribution, is developed to represent the skewness as well as the kurtosis. This distribution is potentially useful for the data that has more skewness and kurtosis. Some statistical properties of the distribution have also been studied.

Key Words - Skewness; Kurtosis; Skew normal; Skew curved normal; Skew generalized normal.

1. Introduction

In statistical literature a general tendency is observed towards more flexible methods in representing the features of data as adequately as possible and in reducing the unrealistic assumptions. For continuous observations within a parametric approach one very important aspect is the normality assumption that underlines the most of the methods of analysis. However, in real life, we are very frequently dealing with phenomena whose outcomes behave in a very non-normal fashion.

To ameliorate such a situation Azzalini (1985) introduced the following lemma that is central to the development of skewed distributions.

Lemma 1. If $f_0$ is a one-dimensional probability density function symmetric about 0, and $G$ is a one-dimensional distribution function such that $G'$ exists and is a density symmetric about 0, then

$$f(z) = 2f_0(z)G\{w(z)\}; \quad -\infty < z < \infty,$$

is a density function for any odd function $w(\cdot)$.
If \( f_0 \) and \( G \) be the density and distribution function of standard normal random variable \( z \) respectively then (1) becomes

\[
f(z) = 2\phi(z) \Phi\{\lambda z\}; \quad -\infty < z < \infty,
\]

where \( w(z) \) is considered as \( \lambda z \), \( \lambda \) is a constant. This is called skew normal distribution that contains the normal distribution as a special case. Thus the skew normal distributions’ family represents an extension of the normal family to which a parameter has been added to regulate the skewness of the distribution.

The introduction of the skew normal distribution led Azzalini (1986), Henze (1986), Azzalini and Dalla-Valle (1996), Branco and Dey (2001), Arnold and Beaver (2002), Arellano-Valle et al. (2004) and others to study the distribution. In an oblique reference to these studies, Matin and Bagui (2008) have studied the small-sample behavior of different estimators of the logistic regression model parameters with skew-normally distributed explanatory variables.

One remarkable property of the skew normal distribution is that as the skewing parameter tends to infinity the distribution behaves like a half normal distribution. In practice, for moderate values of the skewing parameter, nearly all probabilities gather either on the positive or negative numbers that determined by the sign of the skewing parameter (Arellano-Valle et al. 2004). To mitigate such a limitation Arellano-Valle et al. (2004) introduced a class of distributions, skew generalized normal (SGN) distribution, which includes the skew normal distribution and the normal distribution as a special case.

In recent years, not only the skewness but the kurtosis is also of more concern in representing the features of a distribution. In this study a more flexible distribution, extended skew generalized normal (ESGN) distribution, is developed to represent the skewness as well as the kurtosis. This is an extension of the skew generalized normal distribution that includes the SGN, SN and normal distributions. In addition to a skewing parameter it also includes two more extra parameters which can be explained as the parameters that incorporate the kernel of the second and fourth moment of a random variable.

Gomez et al. (2007) developed a general family of skew symmetric distributions which were generated by the cumulative distribution function of the normal distribution. In this family of distribution, it is seen that a significant improvement for asymmetry and kurtosis parameters has been found for the other skew symmetric distributions (skew \( t_3 \)-normal, skew Laplace-normal etc.). However, no improvement has been found for the skew normal-normal distribution over the skew normal distribution for asymmetry and kurtosis parameters. In reality, skew normal-normal distribution and skew-normal distribution both are the same.

In Section 2, we have defined the probability density function of the extended skew generalized normal distribution along with its very simple properties. Section 3 contains some important properties that are related to the