Technical Note: Perturbations on a Stationary Satellite by the Longitude-Dependent Terms in Mars’ Gravitational Field

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Abstract

In analogy with geostationary satellites, an areostationary satellite (a stationary satellite of Mars) will tend to drift in longitude, and in fact, it will tend to oscillate like a pendulum around two stable longitudinal points. In this article we provide a first-order analysis of such motion and provide estimates of the locations of the equilibrium points, the period of libration (130 days for small amplitudes), and the amount of corrective station keeping $\Delta v$ needed to counteract such drift.

Introduction

Geostationary spacecraft, that is, satellites that orbit the Earth in a near-circular, low inclination near-24-h period have proved immensely useful for all types of applications. These types of satellite were first proposed by Konstantin Tsiolkovsky [1] and H. Noordung [2], but their first technical application was proposed by A. C. Clarke as a system of three geostationary satellites to provide worldwide telecommunication coverage [3]. By the early 21st century there were more than 300 geostationary satellites orbiting about the Earth’s equator.

Given their usefulness, it is natural to suppose that in the not-so-distant future there will arise a need (telecommunications, meteorology, and/or other scientific applications; [4–7]) for spacecraft in similar orbits around Mars: this equivalent orbit is called “areostationary.” An areostationary spacecraft would orbit Mars in a zero-eccentricity, zero-inclination orbit such that its orbital period would equal

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2Areo comes from the Greek word for Mars, Ares.
the sidereal rotational period of Mars, 24 h, 37 min. Such spacecraft would, in an ideal case (i.e., pure Keplerian motion), remain completely stationary about 17,000 km above a given point on the surface of Mars, that is, at longitude \( \lambda_s \) (the satellite’s longitude) and latitude \( \phi = 0 \). In practice, however, there would be other forces, such as the gravitational attractions of the Sun and Mars’ two moons, and solar radiation pressure acting on the satellite, such that these will move the satellite away from its ideal orbit.

In this article we are specifically concerned with the perturbative effects that the gravity field of Mars will have on the longitudinal motion (i.e., “drift”) of an areostationary satellite. In the next section “The Gravity Field of Mars” we provide details of the Martian gravitational field. In “Areostationary Satellites” we define and consider the longitudinal acceleration due to the areopotential on areostationary satellites. In “An Approximate Solution” we analyze a simplified version of the longitudinal effects of the gravity field of Mars, and see that its solution corresponds to the pendulum equation. Finally, in “Discussion and Conclusions” we present a discussion and state our conclusions.

The Gravity Field of Mars

The gravity field of Mars (or any other planet) at a point \((r, \lambda, \theta)\) in a body-fixed frame can be written as [8–10]

\[
\Phi(r, \lambda, \theta) = \frac{\mu}{r} \sum_{l=0}^{\infty} \left(\frac{R}{r}\right)^l \sum_{m=0}^{l} P_l^m(\cos \theta) \left[C_{lm}\cos(m\lambda) + S_{lm}\sin(m\lambda)\right]
\]

(1)

where \(\mu = GM\) (\(G\) is the gravitational constant, and \(M\) is the planetary mass); \(R\) is the reference planetary equatorial radius; \(\theta\) is the co-latitude (i.e., \(\theta = 90 - \phi\)); and \(P_l^m(\chi)\) are the associated Legendre functions. The quantities \(C_{lm}, S_{lm}\) are the harmonic coefficients associated with the areopotential of degree \(l\), order \(m\); note that these coefficients must be empirically determined by geodetic means. The functions \(P_l^m(\chi)\) with \(m = 0\) are called zonal harmonics, and these functions are symmetric about the axis of rotation. The functions where \(l = m\) are called the sectorial harmonics, and these can be visualized as when cutting an orange into segments that meet at the poles; we shall pay particular attention to this case later in the article. Finally, all other remaining cases represent the tesseral harmonics, and these functions subdivide the surface of the sphere into a checkerboard pattern [9, 10].

The Perturbing Acceleration

We can re-write the gravitational field as

\[
\Phi(r, \lambda, \theta) = -\frac{\mu}{r} + \Psi
\]

(2)

where the first term (with \(l = 0\)) represents the central Keplerian potential and, by considering the origin of the coordinate system to be located at the center of mass of the system, the \(l = 1\) terms disappear [10].

The acceleration of the spacecraft is given by

\[
\ddot{r} = g = -\nabla \Phi
\]

(3)