ON THE PREDICTION OF THE KURTOSIS OF VELOCITY DERIVATIVES IN A TURBULENT FIELD

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Abstract. The prediction of the values of non-dimensional fourth-order moment (kurtosis) of the velocity derivative in a turbulent field is made under the assumption that the values of kurtosis depend on both the turbulence Reynolds number and the intermittency factor. The method consists of modeling a suitable probability density of the variable in a given turbulence Reynolds number and the intermittency factor.

A crude model of the probability density function is derived, and the numerical calculations based on the model show excellent agreement with many of the experimental data. The analysis shows that the values of kurtosis depend strongly on the intermittency factor, and that depending on the value of the intermittency factor, it is entirely possible to have values of kurtosis as low as five in a flow with a turbulence Reynolds number of 5000.

1. Introduction

It is well accepted that the small-scale structure of turbulence has a spotty or intermittent character, and that the degree of intermittency increases (i.e., the intermittency factor $\gamma$ decreases) with increasing Reynolds number (decreasing turbulent scale). Here, following Kuo and Corrsin (1971), we define the small-scale intermittency, $\gamma$, as the fraction of total space occupied by the fine structure.

Small-scale intermittency is known to have a significant effect on the shape of the probability distribution of velocity gradients (Novikov and Stewart, 1965; Pond and Stewart, 1965; Tennekes, 1973). A consequence of intermittency is the large value of the flatness factor or kurtosis ($K$) of the velocity gradients in high Reynolds number turbulent flows.

Experimental evidence indicates that the kurtosis of velocity gradients is dependent on the associated turbulence Reynolds number $R_\lambda (R_\lambda = (u'\lambda)/\nu; \lambda$ is the Taylor microscale, $u'$ is the turbulence intensity, and $\nu$ is the kinematic viscosity). Corrsin (1962), using a simple model of small-scale structure, predicted that the kurtosis of the first derivative of wind velocity behaves as $R_\lambda^{1.5}$. Later, Tennekes (1968) modified Corrsin's model and found that the kurtosis varied linearly with $R_\lambda$.

Experimental data, however, has failed to confirm either prediction. Experiments by Kuo and Corrsin (1971), in the range of $R_\lambda$ from 12 to 830, indicated that the kurtosis of the first derivative behaves as $R_\lambda^{0.2}$ for $R_\lambda < 200$, and as $R_\lambda^{0.6}$ for $R_\lambda > 500$, with a transition region existing for intermediate $R_\lambda$.

Kolmogorov (1962) and Oboukhov (1962) have suggested that the intermittent nature of kinetic energy dissipation might be represented by a log-normal probability distribution. This idea was later formalized by Gurvich and Yaglom.

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(1967) and has been experimentally verified. Wyngaard and Tennekes (1970) used the log-normality of the dissipation to predict that the first-derivative kurtosis increases as $R_\lambda^{0.5}$. Further analysis, using log-normality behavior, has led to the conclusion that the kurtosis varies as $R_\lambda^{(3/2)\mu}$ (Novikov, 1966; Tennekes, 1973; Frenkiel and Klebanoff, 1975).

Takeuchi (1973) presented experimental results which indicated two widely different kurtosis values (5 and 46) for flows with approximately equal Reynolds numbers. He noted that the differences in kurtosis values obtained was due to the observed differences in the degree of intermittency. Clearly then the kurtosis of the first derivative of velocity is not only a function of the turbulence Reynolds number $R_\lambda$, but also of the intermittency $\gamma$ involved in the process.

Batchelor and Townsend (1949) considered an intermittent variable having a normal probability distribution for a fraction of the time and with zero distribution for the remainder of time. They then concluded that $K \sim 3.0/\gamma$. Kuo and Corrsin (1971), however, measured both $\gamma$ and $K$ in grid-generated turbulence and found that the kurtosis may be better described by $K \sim 7.5/\gamma$. The assumption then of representing an intermittent variable by a normal/zero probability distribution is questionable.

In the current paper, we demonstrate the importance of the degree of intermittency on values of kurtosis. Specifically, we consider the kurtosis of the first derivative of wind velocity. Since the probability distribution is sensitive to intermittency and uniquely determines the kurtosis, we proceed from a prediction of the distribution. It is assumed that the form of the probability distribution is an explicit function of two independent measurable parameters, $\gamma$ and $R_\lambda$.

It should be noted that the purpose of the current paper is twofold. First, it resolves a long-standing problem on Reynolds number dependency of the kurtosis of small-scale turbulence. The analysis also emphasizes, however, the importance of intermittency in obtaining a fundamental understanding of the physics of the small-scale structure of turbulence.

The distributions of small-scale turbulence are highly nonuniform in space and time and have a clearly defined intermittent character. Small-scale turbulence has the tendency to form in individual bunches; the surrounding large-scale disturbances are much smoother. As the scale decreases, and/or the Reynolds number increases, the intermittency effect become more and more clearly defined. Thus, one would expect more uniform or less intermittent character in the distribution of small-scale turbulence as the Reynolds number decreases. (See Monin and Yaglom, 1975, Vol. 2.)

A statistical approach to the problem of turbulence conceals the detailed dynamics of turbulence and only the average effect becomes apparent in the final outcome. In this respect, a model of a statistical quantity such as a probability distribution does not require detailed description of the dynamics and it only is necessary to consider gross features of the flow. The formulation of a model probability density described in the following section is based on such a philosophy.