Optimal investment for insurers

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1. Introduction

It is well known that the success of an insurance company depends not only on its insurance business, but also on how well the company invests its reserve. This paper applies the theoretical results for an integrated risk process (IRP) for an insurance company investing in a bond and in a stock, obtained in Klüppelberg and Kostadinova [3]. A risk measure frequently used in practice – Value-at-Risk (VaR), is defined in the framework of the integrated risk model. We suggest and compare several methods to find an optimal investment strategy, which maximizes the expected wealth of the insurance company, subject to a risk bound. For this we use the VaR as a risk measure.

We first recall the model under consideration. In order to be able to show explicit results, we restrict this paper to the special case where the stock price follows a geometric Brownian motion. More general models based on Lévy processes are treated in [3].

For the insurance business, we assume the classical model. The company starts with some initial capital \(u > 0\) and it receives premiums at a constant rate \(c > 0\). The total claim amount is modeled by a compound Poisson process \(S(t) = \sum_{j=1}^{N(t)} Y_j, \ t \geq 0\). Here \(N = (N(t))_{t \geq 0}\) is a homogeneous Poisson

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process with an intensity of $\lambda > 0$, counting the claims, and $(Y_j)_{j \in \mathbb{N}}$ is a sequence, independent of $N$, of positive iid random variables (rv’s) with a distribution function (df) $F$ and a mean $\mu < \infty$, modeling the claims. Such models are very well studied, see, for example, Embrechts, Klüppelberg and Mikosch [2], Chapter 1.

For the investment we assume a Black-Scholes type market consisting of a bond with a constant interest rate and some stock which follows a geometric Brownian motion. Their respective prices follow the equations

$$X_0(t) = e^{\delta t} \quad \text{and} \quad X_1(t) = e^{B(t)}, \quad t \geq 0,$$

where the constant $\delta > 0$ is the riskless interest rate and $B(t) = \gamma t + \sigma W(t)$, $t \geq 0$, with $\gamma \in \mathbb{R}$, $\sigma > 0$ and $W = (W(t))_{t \geq 0}$ being a standard Brownian motion. The Laplace exponent of $B$ is

$$\varphi(s) = \log E[e^{-sB(1)}] = -\gamma s + \frac{\sigma^2}{2} s^2, \quad s \in \mathbb{R}.$$

We assume that the insurance company invests by using the so-called constant mix strategy. Under such strategy, at each instance of time an initially fixed fraction $\theta \in [0, 1]$ of the wealth is invested in the risky asset and the fraction $1 - \theta$ in the riskless asset. This strategy is dynamic in the sense that it requires a rebalancing of the portfolio at any moment in time depending on the corresponding price changes. This approach is based on self-financing portfolios and, hence, is classical in financial portfolio optimization; see Korn [4], Section 2.1. We call the fraction $\theta$ the investment strategy.

In [3] we derive the integrated risk process (IRP), which models the wealth of the insurance company, for an investment strategy $\theta \in [0, 1]:$

$$(1.1) \quad U_\theta(t) = e^{B_\theta(t)} \left( u + \int_0^t e^{-B_\theta(v)} (c \, dv - dS(v)) \right), \quad t \geq 0.$$

Here $u > 0$ is the initial capital, $c > 0$ is the constant premium rate, $S$ is the total claim amount process, and $B_\theta$ is a Brownian motion with drift and volatility, respectively

$$(1.2) \quad \gamma_\theta = \theta \gamma + (1 - \theta) (\delta + \frac{\sigma^2}{2} \theta) \quad \text{and} \quad \sigma^2_\theta = \theta^2 \sigma^2.$$

In the sequel we will also need the Laplace exponent of $B_\theta$

$$(1.3) \quad \varphi_\theta(s) = \log E[e^{-sB_\theta(1)}] = -\gamma_\theta s + \frac{\sigma^2_\theta}{2} s^2, \quad s \in \mathbb{R}.$$