RECENT WORK ON THE SCOLE MODEL

Walter Littman
University of Minnesota
Minneapolis, MN 55455, USA

1. Introduction.

In a number of papers (see for example [BT]) Balakrishnan and Taylor introduced the "SCOLE" (Spacecraft Control Laboratory Experiment) model for a vibrating flexible mast, which at one end is attached to a spaceship, and at the other end to an antenna reflector. Mathematically, the system consists, essentially, of three uncoupled partial differential equations, two of which are the Euler Bernoulli beam equation while the third is the one dimensional wave equation. At the "left" end "clamped" boundary conditions are imposed. At the "right" end control forces and torques are imposed, yielding complicated non homogeneous boundary conditions which are nonlinear and in which the unknown functions - representing beam deflections and the torsion angle about the beam axis - are coupled.

Two problems present themselves: one is the "open loop" exact controllability of the system: can an initial disturbance - in an appropriated function space - be exactly controlled to rest in a finite time by applying the forces and torques at the right end in an appropriate fashion? The second question is one of closed loop stabilization: Can the (inhomogeneous) control forces and torques at the "right" end be chosen as functions of the velocities and angular velocities at that end in such a way that the energy of the system approaches zero asymptotically as \( t \to \infty \). In that case can this decay be made to exponential? In this note we shall discuss some recent work dealing with the first question.

2. The open loop problem.

In [LM1] the "reduced SCOLE" system is considered, consisting of a single Euler Bernoulli beam equation, arising from the plane motion of a beam.
Consider the mixed problem:

\[
\begin{align*}
  w_{tt} + w_{zzz} &= 0 & 0 \leq z \leq 1, \quad t \geq 0 \\
  w(0, t) &= w_z(t, 0) = 0 \\
  w_{tt} - \beta_1 w_{zzz} &= f_1(t) & \text{at } x = 1 \\
  w(x, 0) &= w_0(x) \\
  w_{t}(x, 0) &= w_1(x) & 0 \leq x \leq 1.
\end{align*}
\]

(Here \(\beta_1 > 0, \beta_2 > 0\)).

The control problem: Given initial conditions \(w_0(x)\) and \(w_1(x)\) (possibly satisfying some compatibility conditions at \(x = 0\)), can we find functions \(f_1(t)\) and \(f_2(t)\) such that the resulting solution of the mixed problem vanishes for \(t \geq T\)?

An answer was given in [LM1]: (Here the \(H\)'s refer to Sobolev spaces)

Given initial data in \(H^6 \times H^4\) on \(0 \leq x \leq 1\), with compatibility conditions

\[
\begin{align*}
  w_0(0) &= w_0'(0) = 0, \\
  w_1(0) &= w_1'(0) = 0 \\
  w_0^{(4)}(0) &= w_0^{(5)}(0) = 0;
\end{align*}
\]

then for each positive duration \(T\), there exist two controllers \(f_1(t)\) and \(f_2(t)\) continuous in \([0, T]\) and \(C^\infty\) on \((0, T]\) such that the corresponding solution, \(w(x, t)\) to the mixed problem vanishes from \(t \geq T\). Furthermore the functions \(f_1(t)\) and \(f_2(t)\) are given by explicit formulas.

Note: In the proof it actually suffices for the initial data to be in \(H^{5\frac{1}{2}} \times H^{3\frac{1}{2}}\).

3. Improvements.

There are several directions in which the method of [LM1] can be extended and improved.

First of all, although the original three dimensional SCOLE system seems much more complicated, the methods of [LM1] encompass essentially all mathematical difficulties, and the exact controllability of the three dimensional model can be achieved by a minor modification of the method. The only difference is that since one of the equations in the full system is a wave equation, the time \(T\) is not arbitrarily small, but is governed by the time it takes to control the one dimensional wave equation.

Secondly, to what extent is the high degree of smoothness of the initial data really necessary? It follows from a result of Triggiani that an initial disturbance assumed only to