SIMPLEX METHOD FOR DYNAMIC LINEAR PROGRAM SOLUTION

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1. INTRODUCTION

In this paper, the extension of the simplex-method, one of the most effective linear programming methods [1,2] to dynamic linear programming [3] is described. The main concept of the static simplex-method—the "global" basis—is replaced by the set of local (for each time period t) bases. It allows us to develop a whole group of finite-step DLP methods: primal, dual and primal-dual methods, each yielding the same solution path as the corresponding static version of the simplex-method. The methods are closely related to the basis factorization approach to DLP problems. We consider a DLP problem in the form:

**Problem 1:** Find a control \( u = \{u(0), \ldots, u(T-1)\} \) and a trajectory \( x = \{x(0), \ldots, x(T)\} \), satisfying the state equation

\[
x(t+1) = A(t)x(t) + B(t)u(t)
\]

with initial condition

\[
x(0) = x^0
\]

and constraints

\[
G(t)x(t) + D(t)u(t) = f(t)
\]

\[
u(t) \geq 0
\]

which maximize the objective function

\[
J_1(u) = a(T)x(T)
\]
Here \( x(t) \) is the \( n \)-vector of state variables; \( u(t) \) is the \( r \)-vector of control variables; \( f(t) \) is the given \( m \)-vector \((t=0,1,...,T-1)\).

This model is flexible enough and allows various extensions and modifications. The results stated below for Problem 1 can be used with minor changes for these extensions and modifications (see Section 3 and \([4]\)).

Along with the primary Problem 1, statement of the dual problem will be necessary \([4]\).

**Problem 2:** Find a dual control \( \lambda = \{\lambda(T-1),...,\lambda(0)\} \) and a dual trajectory \( p = \{p(T),...,p(0)\} \), satisfying the costate equations

\[
\begin{align*}
p(t) &= p(t+1)A(t) - \lambda(t)G(t) \\
&= (t = T-1,...,1,0) 
\end{align*}
\]

with boundary condition

\[
p(T) = a(T)
\]

and constraints

\[
p(t+1)B(t) - \lambda(t)D(t) \leq 0
\]

which minimize the performance index

\[
J_2(\lambda) = p(0)x^0 + \sum_{t=0}^{T-1} \lambda(t)f(t) .
\]

For this pair of dual problems the conventional duality realizations hold \([4]\).