"Reaction of continuous dynamic systems with complex form under time-space random fields".

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Introduction.

Correlational and spectral analysis methods are widely applicable within the scope of examining reactions of discrete and continuous dynamic systems subjected to random excitations. [2], [5], [6], [7], [8], [9], [10], [11], [12], [13], [15], [18], [19], [20]. They examine interrelations between the probabilistic characteristics (average value), the correlative function of random excitation processes conveniently referred to as the "input" and random processes describing the state of the system referred to as "output". In the case of continuous dynamic systems the random fields constitute the random processes. It is the aim of this work to present a certain new mathematical method for the determination of probabilistic characteristics of continuous dynamic systems with a complex form subject to non-stationary excitations by random fields. These problems arise in various technical problems e.g. by examining starting vibrations of air and rocket constructions. The demonstration of the methods has been carried out for the case of vibrating plates having a complex form and fixed stiff along the whole edge. The method discussed due to the applications of certain special functions introduced by V.L. Rvatschev [17] called the R-functions enabled solutions of the problem in the form of closed analytical formulas. Applying the R-functions one can obtain the equation of the outline of the edge of the area. This area describes practically free geometry. The characteristic feature of the R-function is that each of them corresponds to a definite logical function, the arguments of the latter are two discrete values 1 and 0. This quality enables to apply the contemporary methods of the algebra of logic to the solution of the boundary problems of mathematical physics in the fields of complex form. The logic function which corresponds to the R-function takes the value 1 when the point under investigation lies within the area or on its edge and the value 0 when it is outside the area. A free area with complex form can be set together by means of the multipli-
City operations of addition, multiplication etc. upon the areas with simple form the equations of which are known. In the case of the R-functions the multiplicity operations will correspond then to the logical operations such as the OP /alternative/ operation and the AND /conjunctive/ operation. Thus the logical functions corresponding to the R-functions will possess in itself a certain coded information about the change - over /switch over, commutation/ of the sign in the equations describing the edge of the areas with complex form.

1. The formulation of the problem.

The stochastic boundary problems of mathematical physics, connected with linear continuous dynamic systems, can be written in the following operational form:

\[ A \mathbf{u} = f(\mathbf{x}, t, \gamma) \] (1)
\[ B_j \mathbf{u}\big|_{\partial \Omega} = 0 \] (2)
\[ \mathbf{D}_t \mathbf{u}\big|_{t=t_0} = 0 \] (3)
in the area \( \Omega \subset \mathbb{R}^n \)
\[ \partial \Omega \] - the edge of the area \( \Omega \)
\( f/\mathbf{x}, t, \gamma / \) - the exciting measurable random fields induced by probabilistic space \( /\Gamma, F, P/ \).
\( \gamma, \delta \) is the set of elementary sentences, \( F \) is the \( \sigma \) - algebra, \( P \) - is the probabilistic measure in the probabilistic space \( /\Gamma, F, P/ \).
\( A, B_j \) - are the linear differential operators in the equation / 1 / and in the boundary conditions / 2 /.
\( \mathbf{D}_t^k \) - the multi indicatory symbol of the differentiation of the relative time \( t \).

In the case of the linear, continuous dynamic system which random vibrate the operator \( A \) appears most often in the shape of:

\[ A \mathbf{u} = a_0 \frac{\partial^2 \mathbf{u}}{\partial t^2} + a_1 \frac{\partial \mathbf{u}}{\partial t} + L \mathbf{u} \] (4)

where

\[ a_0, a_1 \] - are certain positive constants.

\[ L \mathbf{u} = \sum_{\mid \mathbf{p} \mid \mid q \mid \leq m} (-1)^{\mid \mathbf{p} \mid} \mathbf{D}^\mathbf{p} (a_{pq}(\mathbf{x}) \mathbf{D}^\mathbf{q} \mathbf{u}) \] (5)

\[ a_{pq}(\mathbf{x}) \in \mathcal{C}^\infty(\Omega) \]