Statement of results. In this note we consider the solution of the problem

\[ u_{tt} + 2\beta u_t + [\alpha + \epsilon N(t)]u = c^2 \Delta u \]

\[ u = 0 \text{ on } \partial D \]

\[ u(x;0) = f_1(x) \quad u_t(x;0) = f_2(x) \]

where \( \alpha > 0, \beta > 0, c > 0, \epsilon > 0 \). \([N(t), t \geq 0]\) is a noise process with mean zero, which may be either the formal derivative of a Wiener process or a centered function of an ergodic Markov process. In the first case the equation is interpreted as an integral equation of the Itô type. \( D \) is a smoothly bounded domain in Euclidean space with Dirichlet eigenvalues \( 0 < \mu_1 < \ldots < \mu_n < \ldots \) and normalized eigenfunctions \( \phi_n \) satisfying \( \Delta \phi_n + \mu_n \phi_n = 0 \) in \( D \) with \( \phi_n = 0 \) on the boundary \( \partial D \) and \( \int_D \phi_n^2 \, dx = 1 \).

The energy of the solution \( u \) is defined by

\[ E(t) = \frac{1}{2} \int_D [u_t^2 + c^2 |\nabla u|^2 + \alpha u^2] \, dx \]

We have the following results.

**Theorem 1.** Suppose that \( \beta^2 - \alpha > \mu_1 c^2 \). Then there exists the limit

\[ \lambda(\epsilon) = \lim_{t \to \infty} t^{-1} \log E(t) \]

and when \( \epsilon \downarrow 0 \) we have

\[ \lambda(\epsilon) = -2\beta + 2\sqrt{\beta^2 - \alpha - \mu_1 c^2} + O(\epsilon). \]

**Theorem 2.** Suppose that \( \beta^2 - \alpha \leq \mu_1 c^2 \). Then there exists

\[ \bar{\lambda}(\epsilon) = \limsup_{t \to \infty} t^{-1} \log E(t) \]

and we have \( \limsup_{\epsilon \downarrow 0} \bar{\lambda}(\epsilon) \leq -2\beta \). (This includes the "underdamped case" \( \beta^2 - \alpha \leq 0 \).

We can also obtain results in the case of continuous spectrum. The initial-boundary-value problem for the wave equation (1.1) in the full Euclidean space \( \mathbb{R}^d \) with the condition that the solutions have finite energy, defined by

\[ E(t) = \frac{1}{2} \int_{\mathbb{R}^d} [u_t^2 + c^2 |\nabla u|^2 + \alpha u^2] \, dx. \]

The following theorems are obtained.
Theorem 3. Suppose that $\beta^2 - \alpha > 0$. Then there exists the limit

$$\lambda(\varepsilon) = \lim_{t \to \infty} t^{-1} \log E(t)$$

and when $\varepsilon \downarrow 0$ we have

$$\lambda(\varepsilon) = -2\beta + 2\sqrt{\beta^2 - \alpha} + O(\varepsilon)$$

Theorem 4. Suppose that $\beta^2 - \alpha \leq 0$. Then there exists

$$\bar{\lambda}(\varepsilon) = \limsup_{t \to \infty} t^{-1} \log E(t)$$

and we have $\limsup_{t \to \infty} \bar{\lambda}(\varepsilon) \leq -2\beta$.

2. Proof in white noise case

We introduce the Fourier components $u_n(t)$ through the eigenfunction expansion

$$u(x; t) = \sum_{n=1}^{\infty} u_n(t) \phi_n(x).$$

The stochastic wave equation translates into the system of ordinary differential equations

$$u_n'' + 2\beta u_n' + [\alpha + \mu_n c^2 + \varepsilon N(t)]u_n = 0 \quad n = 1, 2, \ldots$$

and the energy is computed as

$$E(t) = \frac{1}{2} \sum_{n=1}^{\infty} [(u_n')^2 + (\alpha + \mu_n c^2)u_n^2].$$

Define $v_n(t) = u_n(t)e^{\beta t}$. The equation for $u_n$ translates into

$$v_n'' + [\alpha - \beta^2 + \mu_n c^2 + \varepsilon N(t)]v_n = 0$$

and the energy is

$$E(t) = e^{-2\beta t} \sum_{n=1}^{\infty} [(v_n')^2 + (\alpha + \mu_n c^2)v_n^2].$$

This is decomposed as $E(t) = E_1(t) + E_2(t)$ where $E_1(t)$ is the sum over those indices $n$ for which $\mu_n c^2 \leq \sqrt{\beta^2 - \alpha}$ and $E_2(t)$ is the sum over the remaining indices. We will show that

Lemma 1. $\limsup_{t \to \infty} t^{-1} \log E_2(t) \leq -2\beta^2$

For the proof introduce the functional

$$\tilde{E}_2(t) := \frac{1}{2} \sum_{n=N_0}^{\infty} [(v_n')^2 + (\alpha - \beta^2 + \mu_n c^2)]$$

where $N_0$ is the first integer $n$ for which $\mu_n c^2 > \beta^2 - \alpha$. By direct calculation we have

$$\tilde{E}_2(t) = \sum_{n=N_0}^{\infty} \varepsilon N(t) v_n(t) u_n'(t).$$