SHAPE OPTIMIZATION IN CONTACT PROBLEMS. 1. DESIGN OF AN ELASTIC BODY.
2. DESIGN OF AN ELASTIC PERFECTLY PLASTIC BODY

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Abstract The optimal shape design of a two dimensional body on a rigid foundation is analyzed. The problem is how to find the boundary part of the body where the unilateral boundary conditions are assumed in such a way that a certain energy integral (total potential energy, for example) will be minimized. It is assumed that the material of the body is elastic. Some remarks will be given concerning the design of an elastic perfectly plastic body. Numerical examples will be given.

1. INTRODUCTION

During the last 15 years, optimization theory has been developed for the optimal shape design of many structural and mechanical systems. The goal is to solve an optimal shape control problem which involves optimization of a system modelled by partial differential equations with respect to some geometric element of the system (shape, thickness, etc.). Mathematical theory of such kind of problems including the theory of their approximation by FEM has been developed during the last ten years (see [1-14] and bibliography therein).

The main part of our contribution (chapters 2-5) deals with a two-dimensional elastic body on a rigid foundation. The influence between the body and the support will be taken into account by applying the model with a given friction. In our shape optimization problem the contact boundary of the body must be redesigned in such a way that the total potential energy of the system will be minimized.

The design problem is formulated in chapter 2 and in chapter 3 discretization by a finite element method is presented. Chapter 4 is devoted to the sensitivity analysis. We give a formula for the gradient of the criterion function with respect to design variables. The gradient information is necessary for the application of efficient nonlinear programming algorithms for solving numerically the problem in question. In chapter 5 a numerical example is given. In the last part of this contribution an optimal shape design problem for an elastic perfectly plastic body obeying Henky's law is given.

For contributions to optimal shape design problems in elasticity (without friction) and their numerical solutions we refer to [1,4,5,7,8] and references therein.

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2. THE SHAPE OPTIMIZATION PROBLEM

Let us consider a two-dimensional elastic body \( \Omega = \Omega(a) \subset \mathbb{R}^2 \) having the following geometrical structure

\[
\Omega(a) = \{(x_1, x_2) \in \mathbb{R}^2 \mid a < x_1 < b, \ 0 \leq a(x_1) < x_2 < \gamma\},
\]

with given constants \( a, b, \gamma > 0 \) and a function \( a \in C^1,1([a,b]) \);

\[
\partial \Omega(a) = \Gamma_D \cup \Gamma_F \cup \Gamma_C(a), \quad \Gamma_D \neq \phi,
\]

\[
\Gamma_C(a) = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_2 = a(x_1), \ x_1 \in (a,b)\}.
\]

A possible partition of \( \partial \Omega(a) \) is given in Fig. 2.1.

![Fig. 2.1 \( \Omega(a) \)](image)

The shape of the contact surface \( \Gamma_C(a) \) is defined by a control parameter \( a \) from the set \( \mathcal{U}_{\text{ad}} \) of admissible controls

\[
\mathcal{U}_{\text{ad}} = \{a \in C^1,1([a,b]) \mid 0 \leq a(x_1) \leq C_0 < \gamma, \ |a'(x_1)| \leq C_1, \ |a''(x_1)| \leq C_2, \ \forall x_1 \in [a,b], \ \text{meas} \ (\Omega(a)) = C_3\},
\]

where \( C_0, C_1, C_2, C_3 \) are positive constants chosen in such a way that \( \mathcal{U}_{\text{ad}} \neq \phi \).

Suppose that the body \( \Omega(a) \) is unilaterally supported by a rigid foundation (here by the set \( \{(x_1, x_2) \in \mathbb{R}^2 \mid x_2 \leq 0\} \) and subjected to a body force \( F = (F_1, F_2) \) and to a surface traction \( F = (F_1', F_2') \) on \( \Gamma_F \).

In the classical formulation of contact problems one looks for a displacement field \( u = u(a) = (u_1(a), u_2(a)) \) satisfying the equilibrium equations (the dependence of \( u \) on \( a \) is emphasized by writing \( u = u(a) \))

1) Throughout the paper, the summation convention is used.