INSTABILITIES IN ADAPTIVE CONTROL SYSTEMS

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Abstract

An exposition of some of the results in continuous-time adaptive control systems in stability is given. We study instabilities that arise due to: high adaptation gain and/or large reference signals, interplay between unmodeled dynamics and output disturbances and crosscoupling in decentralized controllers. In all cases simple worked examples are included to illustrate the phenomena. In the disturbance-free case, the causes of instability are rigorously explained via instability theorems. The basic purpose of the paper is to establish criteria for instability, however, conditions for stability are also formulated concurrently.

I. Introduction

A significant proportion of the recent research in adaptive control theory has been devoted to the problem of stability robustness. This interest stems from the fact that robustness of stability is often the main concern in control, rather than optimality with respect to a (sometimes somewhat arbitrary) mathematical performance criteria. In other words, the main issue in many control problems is to guarantee that the plant will operate in the neighborhood of a desired operating point in spite of model-process mismatch.

Accomplishment of the latter task in adaptive control is hampered by two factors: 1) It is now widely recognized\(^2\) \(^3\) that intrinsic to the standard adaptation laws are mechanisms that may trigger instability in the presence of nonidealities. These instabilities occur only when adaptation is on\(^3\). This in contrast with the better understood linear instabilities for which the linear feedback loop is unstable even when the adaptation is switched off. See\(^3\) for further details. 2) To the best of the present authors' knowledge, except for the case of slow adaptation, where sufficient conditions for local instability are given, see e.g.\(^4\), there are no available results on "how close" to necessity are the given sufficient conditions. As pointed out in\(^1\)\(^2\), determination of a "nearness to instability" measure is of utmost importance to evaluate the robustness of a given feedback control system.

Must of the reported results on robustness of adaptive controllers focus on the model mismatch due to unmodeled dynamics and/or output disturbances. Two avenues of research have been pursued: the derivation of conditions for stability, possibly with
modified laws, and the explanation of the instability mechanisms arising in the adaptation loop. Considerable progress has been made along both lines. Conditions for global stability for continuous and discrete-time adaptive controllers have been established. The proofs are centered on the assumption of the existence of a parametrization for the closed-loop system such that its transfer function is strictly positive real (SPR). The difficulty encountered in removing, in a global context, the rather unphysical and unrobust SPR condition has moved several researchers to consider the local stability problem. In this formulation, the SPR condition has been relaxed using exponential stability and averaging techniques. Both approaches highlight the importance of satisfying certain excitation conditions on the command signal. Further contributions to the understanding of the interplay between excitation and unmodeled dynamics have been reported in. For an excellent report on the state of the art of the subject see.

Apparently, the first rigorous analysis of instability in adaptive systems was obtained in, where the case of slow adaptation is considered. The conditions derived in the paper have an intuitively interpretative frequency interpretation in terms of the process transfer function and the regressor richness which define in this way a sharp stability/instability boundary. The technique of has been applied to study the instability mechanisms due to the existence of crosscoupling terms in decentralized control schemes and to the presence of an unmodeled pure delay in the process. Using generalized Floquet theory a stability/instability boundary for systems with non-necessarily low adaptation gain is also derived in.

Instabilities in control schemes which contain no feedback except the implicit feedback that comes through the adaptation mechanism have been analyzed in. The study of such schemes allows, on one hand to utilise the powerful absolute stability analysis techniques (see e.g.) in a global framework, and on the other hand to isolate the cause for instability that rests solely with the adaptation loop. Whatever their practical significance, which is discussed in, the fact that they belong to a class of systems for which fairly complete stability/instability results exist, makes their study theoretically appealing to understand in how far the assumptions for global stability are fundamental and how to modify the adaptive laws to enhance robustness.

In this paper we study instabilities that arise due to the following reasons: high adaptation gains and/or large reference signals, interplay between unmodeled dynamics and output disturbances and crosscoupling in decentralized controllers. The purpose of the paper, which surveys some existing results, is twofold. Firstly, to exhibit through simple examples the instabilities mentioned above. Secondly, to rigorously explain the causes of instability. The latter is accomplished by means of stability/instability theorems for the example in consideration, and when possible, for the general case.