ABSTRACT. A new class of dynamic models for stationary time series is presented. It is a natural dynamic generalization of the well-known Factor Analysis Model widely used in Statistics. Factor Analysis models of time series are also related to dynamic Errors-in-Variables models discussed in the recent literature. They provide simple mathematical schemes for the identification of multivariate time series which avoid the unjustified introduction of causality relations among the variables, as for example subsumed by conventional ARMAX models.

1. INTRODUCTION

Recent papers by KALMAN (/8/,/9/,/10/) have stimulated the study of dynamical models for time series identification which do not a priori impose causality relations among the variables. Dynamic Errors-in-Variables (E.I.V.) models have in particular been object of much interest (see e.g. /1/,/2/,/4/) in this framework.

In this paper we shall study a related class of mathematical descriptions, which explicitly introduce a "dynamical interaction" variable x summarizing all cross correlation existing between two given random processes $Y_1(t)$, $Y_2(t)$. This auxiliary variable plays a role similar to the state variable in Systems Theory and, once it is eliminated, E.I.V. models appear as "external behaviour". The models introduced here reduce to linear Factor Analysis models in the static case. Motivations for considering these objects as "natural" (unprejudiced) descriptions of interacting variables are provided in /13/. For reasons of space limitation all technical details and proofs will be skipped. A more detailed analysis than the one which is possible to give here will appear in /14/.

DEFINITION 1.1

A dynamic Factor Analysis model with external variables the jointly stationary zero-mean vector processes $(Y_1(t))$ and $(Y_2(t))$, is a linear relation of the form

\[ Y_1(t) = A_1(z)x(t) + W_1(t), \]
\[ Y_2(t) = A_2(z)x(t) + W_2(t), \]
where $A_1(z)$ and $A_2(z)$ are transfer matrices of dimensions $m_1 \times n$ and $m_2 \times n$ and

$$\{x(t)\}, \{w_1(t)\}, \{w_2(t)\}$$

are zero-mean stationary processes of dimensions $n, m_1, m_2$, which are pairwise uncorrelated, i.e.

$$\{w_1(t)\} \perp \{x(t)\} \perp \{w_2(t)\}. \quad (1.2)$$

In general, $A_1$ and $A_2$ need not be causal. The process $\{x(t)\}$ will be referred to as the factor process of the model. A dynamic Factor Analysis (F.A.) model will be called rational if $A_1, A_2$ are rational matrices and $\{x(t)\}$ has rational spectrum.

In this paper we shall present a first rudimentary analysis of the model (1.1). The main questions one would like to answer concern i) the representability of an arbitrary $m$-dimensional stationary process $\{y(t)\}$ (with $y(t)$ given in partitioned form as $y(t)=[y_1(t)\ y_2(t)]'$, with $y_k(t) \in \mathbb{R}^{n_k}, k=1,2$ and $m_1+m_2=m$) by models of the type (1.1); ii) the equivalence of representations, i.e. when do different representations describe the same process $\{y(t)\}$; iii) the study of the "external behaviour" which is obtained once the variable $x$ is eliminated from the model (1.1); iv) finding a natural notion of minimalit y and characterizations of minimal models; v) parametrizations and canonical forms in the rational case and, above all, vi) the use of Factor Analysis models in Statistical Inference (i.e. identification).

This is quite a large program and only a few of these aspects will be touched upon in this paper. Others (especially the last two mentioned above) still need more research and will be discussed elsewhere.

2. DYNAMIC FACTOR ANALYSIS MODELS

The factor space $X$ of the model (1.1) is the Hilbert space of random variables (/15/) generated by the factor process,

$$X: = \overline{\text{span}\{\cdot'x(t); \ a \in \mathbb{R}^n, t \in \mathbb{Z}\}}. \quad (2.1)$$

Note that $X$ is a doubly invariant (or stationary) subspace for the shift group $U$

$$(U \cdot n(t) := n(t+1))$$

attached to the (jointly) stationary processes $x, w_1, w_2$ defining the model, i.e. $U^tX=X$ for all $t \in \mathbb{Z}$. Recall that the multiplicity of a doubly invariant subspace $X$ is the cardinality of any minimal generating set, i.e. the smallest $n$ for which one can find random variables $\{\xi_1, \ldots, \xi_n\}$ in $X$ such that the vector space generated by $\{U^t_{\xi_1} ; i=1, \ldots, n, t \in \mathbb{Z}\}$ is dense in $X$. The process $\{\xi(t)\}$ with

$$\xi_k(t) := U^t_{\xi_k}, \quad i=1, \ldots, n$$

is called a generating process of $X$. We shall adhere to the convention of considering only F.A. models in which $\{x(t)\}$ is a minimal generating process for the factor space $X$. The multiplicity of $X$ will then always coincide with the dimension, $n$, of $x$. Two F.A. models which differ by a change of (minimal) generators in $X$ will be called equivalent. Two equivalent models have the same $\{w_k(t)\}$ processes (for $k=1,2$), the same factor space $X$ and transfer matrices and factor processes related by