AN INTERACTIVE PROCEDURE BASED ON
THE INSCRIBED ELLIPSOID METHOD

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Interactive procedures for multicriterion decision problems are traditional objects of study in design of decision support systems for economic control and management. This paper will consider a linear multiple criteria problem

$$\max \succ y$$ \quad (1)

subject to

$$y = Cx, \quad Ax \leq a, \quad x \geq 0$$ \quad (2)

where $C, A$ and $a$ are some matrices and a vector of appropriate dimensions, $x \in \mathbb{R}^n$ is the vector of variables and $y \in \mathbb{R}^k$ is the criterion vector. The constraints (2) of the model are given explicitly and the only nonformalized component of the multicriterion decision problem (1),(2) is the decision-maker's preference relation $\succ$ in (1). It is worth mentioning that in typical problems $n \gg k$, and the number $k$ of criteria rarely exceeds 10.

To proceed further we state some assumptions on the decision maker's preference relation $\succ$ in the criterion space $\mathbb{R}^k$ and give some definitions.

$\varepsilon$-optimal solutions

Let $X=\{ x \in \mathbb{R}^n \mid Ax \leq a, \quad x \geq 0 \}$ be the feasible set of the problem and $Y=\mathbb{R}^k$ be the reachability domain in the criterion space.

Throughout this paper we assume that decision maker's preference relation $\succ$ can be described as a quasiorder in $\mathbb{R}^k$ (i.e. this binary relation is reflexive, transitive and complete). As usual, the vector $y^* \in Y$ is called the most preferable in $Y$, if $y^* \succ y$ for all $y \in Y$. 
i.e. if \( y^* \) dominates the reachability domain. We will also refer to \( y^* \) as an optimal estimate and denote by \( Y^* \subseteq Y \) the set of all the optimal estimates in the reachability domain.

Remark. It is well known [6], that \( Y^* \) is a nonempty compact set provided that \( Y \) is nonempty and compact and preference relation \( \succ \) is continuous.

We now proceed to the notion of \( \varepsilon \)-optimality. Let us fix some metric \( \rho(.,.) \) in the criteria space \( \mathbb{R}^k \) and an arbitrary positive constant \( \varepsilon \).

We say that a criterion vector \( z \) \( \varepsilon \)-dominates a criterion vector \( y \) and write \( z \succ_{\varepsilon} y \) if there exist sufficiently close vectors \( y' \) and \( z' \)

\[
\rho(y, y') \leq \varepsilon, \quad \rho(z, z') \leq \varepsilon
\]
such that \( z' \succ y' \).

A vector \( y^*_\varepsilon \in Y \) is called an \( \varepsilon \)-optimal estimate for the problem, if \( y^*_\varepsilon \) \( \varepsilon \)-dominates the reachability domain \( Y \).

Remark. It is easy to see that the set \( Y^*_\varepsilon \) of all the \( \varepsilon \)-optimal estimates is nonempty for any nonempty compact reachability domain \( Y \) (without the continuity assumption on \( \succ \)). If \( \succ \) is continuous, then \( Y^*_\varepsilon \rightarrow Y^* \) as \( \varepsilon \rightarrow 0 \).

In what follows we assume that the reachability domain \( Y \) in (1),(2) is nonempty and bounded (compact) and consider the metric

\[
\rho(y, z) = \max_{1 \leq i \leq k} \left| \frac{y^i - z^i}{\Delta^i} \right|, \quad (3)
\]

where

\[
\Delta^i = \max_{y \in Y} y^i - \min_{y \in Y} y^i
\]
is the range of the \( i \)-th criterion on \( Y \). This metric naturally arises in numerous engineering and economic applications.

Separable preference relations

The last and the most crucial assumption on the decision maker's preference relation \( \succ \), the separability assumption. This assumption is