Analogical Reasoning
and
Proof Discovery\(^1\)

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Abstract

We introduce preliminary research on the problem of applying analogical reasoning to proof discovery. In our approach, the proof of one theorem is used to guide the proof of a similar theorem by suggesting analogous steps. When a step suggested by a guiding proof cannot be applied, actions are taken to bring the proofs back into correspondence, often by adding intermediate steps. Taking this approach, we have implemented a natural deduction prover which exploits analogical reasoning and has yielded some promising results in the domain of Real Analysis. We present some of these results, which include a proof of the convergence of the product of convergent sequences, using an analogous proof for the sum of convergent sequences. We also include the timing results of one experiment in which our prover's performance was compared with and without the use of analogy.

1 Motivation

Much of the work that has been done in the area of mechanical problem solving can be placed near the extremes of a spectrum. At one end, much research in Automated Theorem Proving has focused on solving difficult problems by developing efficient algorithms and fast search strategies. Many of these strategies involve an easily characterized, restricted search through a state space generated by applications of a small set of inference rules. The Set of Support strategy for resolution is an example. Such strategies are often justified (or even motivated) by a completeness proof. At the other end of the spectrum, much problem solving research in Artificial Intelligence has attempted to incorporate more human-like problem solving behavior. The results of these efforts tend to be knowledge-intensive. The field of Expert Systems provides many examples.

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A potential limitation of the first focus is that simple search strategies may not be adequate for solving the difficult problems to which humans apply a wealth of knowledge and experience. Yet most of the research motivated by the knowledge-oriented approach is applied to domains which are less complicated than the domains to which many theorem provers are applied. It is not clear that the methods which produced many successful expert systems will yield the same success when applied to difficult domains of mathematics. Thus we are anxious to experiment with applying knowledge-intensive methods in automating proof discovery. In our current research, we have emphasized the use of analogical reasoning. For some examples of related research efforts in the application of analogical reasoning to problem solving, see [7], [9], [10], and [13].

In this paper, we will first describe how we apply analogical reasoning to theorem proving. We will then present the results of some experiments with a natural deduction implementation in the domain of Real Analysis. We will briefly discuss current and future work, and finally conclude with lessons we have learned from the research conducted so far. This paper is intended as an overview of ongoing research. Readers interested in more technical details of our implementations are referred to [4], [5], and [6].

2 Proof Discovery using a Guiding Proof

A problem solver which takes full advantage of analogical reasoning could include several facilities. Given a target problem to solve, it could retrieve a similar problem and associated solution from a large knowledge base. It could then find a mapping between the guiding problem and the target problem which summarizes how the terms of the target theorem correspond to the terms of the guiding theorem. Using the guiding solution and the mapping between the two problems, it could discover a solution to the target problem. Finally, it could store the new problem/solution pair in the knowledge base for use in solving later problems. Although we are ultimately interested in all these aspects of analogical reasoning, the research presented in this paper has been directed primarily toward the task of discovering a proof for a target theorem by following the proof for an analogous theorem. Thus our implemented system requires that the user provide the guiding theorem and proof as well as the mapping between the guiding and target theorems.

In [4], we describe a resolution based implementation which formed the basis for our first experiments. Our current implementation uses natural deduction and is described more fully in [5] and [6]. Both implementations use chaining and variable elimination (described in [2]), and both have the capability of fetching lemmas from a lemma base when necessary for completing a proof. In the following subsections, we describe how our current implementation follows a guiding proof, how it recovers when an analogy breaks down, and how it attempts to complete a proof when the