The theory of partial algebras seems to be a natural framework for a study of algebraic properties of programs. However, this theory is not the most explored part of universal algebra and it may happen that it does not always offer tools supporting fruitfully these investigations.

In our categorical researches we have developed special methods and techniques introduced mainly for investigation of "categories of mixed structures (i.e. neither purely algebraic, nor purely topological). Some classes of partial algebras, called weak varieties, are within the scope of the categorical theory developed. In this note we are going to present applications of these categorical methods and techniques in studying algebraic properties of programs.

An intuitive meaning of a program in a class of algebraic systems (or partial algebras) is clear; we start with fundamental operations and term operations and then we construct programs using "branches", "loops", "restrictions" and composition of programs.

In this note, however, we do not follow this scheme of construction of programs. Instead, we proceed as follows: first we define the notion of an implicit operation in a weak variety which is, from semantical point of view, a natural generalization of the concept of a term operation. We give a structural description of implicit operations referring to the categorical concept of a free spectrum of a weak variety. Next, using this structural description, we define the class of "programs" as a subclass of implicit operations.

The defined class is greater then the class of programs in the intuitive sense. However, we prefer this approach because the defined class has a nice structure which makes possible a precise analysis of relationship between programs. Namely, we show that programs with a fixed arity form a sheaf, uniquely determined by the class of partial algebras considered.

I. BASIC CONCEPTS

Let \( \Omega \) denote an arbitrary but fixed finitary type. By \( \text{PAlg}\Omega \) we denote the category of partial \( \Omega \)-algebras and their homomorphisms.
Recall that a homomorphism of partial \( \Omega \)-algebras \( h:(A,(q^A))\to(B,(q^B)) \) is strong provided for every operation symbol \( q \) and \( a \in A^n \), whenever \( q^B(ha) \) is defined then \( q^A(a) \) is defined (and then \( hq^A(a) = q^B(ha) \)) ([3]).

**Definition 1.** ([6]) A class \( V \) of partial \( \Omega \)-algebras (as well as the full subcategory of \( \text{Palg} \Omega \) it determines) is called a weak variety provided it satisfies the following:

i.) whenever \( e:A\to B \) is surjective and strong and \( A \in V \), then \( B \in V \),

ii.) if \( (m^I_i:A\to A^I_i; i \in I) \) is a jointly monomorphic family containing at least one strong homomorphism and every \( A^I_i \in V \), then \( A \in V \).

We call a weak variety elementary iff it is closed under formation of ultraproducts. For a first order description of elementary weak varieties we refer the reader to [6].

If a weak variety \( V \) consists of total \( \Omega \)-algebras only, then \( V \) is a variety (i.e. an equationally definable class). If \( V \) is closed under products, then \( V \) is an ece-variety in the sense of P. Burmeister ([1]).

Every relation may be considered as a partial projection: for a given relation \( r \subseteq A^n \) we define \( p_r:A^n\to A \), where \( \text{dom}(p_r) = r \) and \( p_r \) is a restriction of the first-component projection. Thus relational systems as well as algebraic systems may be regarded as partial algebras. In particular, every universal class of algebraic systems may be regarded as a weak variety of partial algebras of a suitably defined type ([6]).

Also many sorted (partial) algebras may be regarded as partial algebras.

**EXAMPLE.** Let \( \hat{\Omega} \) be a two sorted signature with one binary operation symbol \( q \) of the scheme \((0,1;1)((4)). \) Then the category of all total \( \hat{\Omega} \)-algebras is isomorphic to the weak variety \( V \) of partial \( \hat{\Omega} \)-algebras where \( \hat{\Omega} \) consists of a binary symbol \( q \) and two unary symbols \( s, r \), while \( V \) consists of partial \( \hat{\Omega} \)-algebras satisfying the following (universal) formulas:

\[
\exists r(x) \lor \exists s(x), \quad \neg(\exists r(x) \land \exists s(x)),
\]

\[
\exists r(x) \land \exists s(y) \lor \exists q(x,y), \quad \exists q(x,y) \lor \exists s(q(x,y)).
\]

(Here and in what follows, for a given term \( t \), the symbol \( \exists t(x_1,\ldots,x_n) \) is an abbreviation of a first order formula such that for any partial algebra \( A \) and a valuation \( h \) of variables \( x_1,\ldots,x_n \) in \( A \), \( A \) satisfies \( \exists t(x_1,\ldots,x_n) \) at \( h \) (\( A \models \exists t(x_1,\ldots,x_n)[h] \) in symbols) iff the term \( t(hx_1,\ldots,hx_n) \) is defined in \( A \) ([6]). These formulas are called existential atomic formulas).

We hope these remarks show that the concept of a weak variety covers a wide class of categories of algebraic structures investigated in the algebraic theory of programs.

**Definition 2.** By an \( n \)-ary implicit operation in a weak variety \( V \subseteq \text{Palg} \Omega \) we mean a family \( \phi \) of partial functions,