Dataflow Networks are Fibrations

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Abstract

Dataflow networks are a paradigm for concurrent computation in which a collection of concurrently and asynchronously executing processes communicate by sending messages over FIFO message channels. In a previous paper, we showed that dataflow networks could be represented as certain spans in a category of automata, or more abstractly, in a category of domains, and we identified some universal properties of various operations for building networks from components. Not all spans corresponded to dataflow processes, and we raised the question of what might be an appropriate categorical characterization of those spans that are “dataflow-like.” In this paper, we answer this question by obtaining a characterization of the dataflow-like spans as split right fibrations, either in a 2-category of automata or a 2-category of domains. This characterization makes use of the theory of fibrations in a 2-category developed by Street. In that theory, the split right fibrations are the algebras of a certain doctrine (or 2-monad) $R$ on a category of spans. For the 2-categories we consider, $R$ has a simple interpretation as an “input buffering” construction.

1 Introduction

Dataflow networks [4, 5] are a paradigm for concurrent computation in which a collection of concurrently and asynchronously executing processes communicate by sending messages over FIFO message channels. 

Determinate dataflow networks compute continuous functions from input message histories to output message histories, and have a well-understood theory. Less developed is the theory of indeterminate or non-functional networks. These more general networks are especially interesting because they exhibit both concurrency and indeterminacy, and insight gained from their study will likely contribute to a better overall understanding of these two concepts.

This paper is part of a research program aimed at finding the correct algebraic setting for the study of indeterminate dataflow networks. We wish to view dataflow networks as the elements of an algebra whose operations represent ways to build networks from

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components, and we would like to understand fully the notions of behavioral equivalence that are appropriate in this context. For some time, we have been studying a particular automata-theoretic model for dataflow networks, in an attempt to identify whatever useful algebraic structure might be present. Based on the progress we have made so far [8, 9, 10, 13], a general structure appears to be emerging. However, all is not yet completely clear, and it continues to be difficult to identify and separate the important structure from the incidental artifacts of the model.

In a previous paper [9] we showed that a dataflow network with input "ports" \( X \) and output ports \( Y \) could be represented as a span from \( FX \) to \( FY \) (i.e. a diagram \( FY \leftarrow A \rightarrow FX \)) in a finitely complete category \( \text{Auto} \) of concurrent automata. Here \( F \) is a suitable functor that associates "objects of inputs" \( FX \) and \( FY \) in \( \text{Auto} \) with finite sets of ports \( X \) and \( Y \). We showed that various constructions, corresponding intuitively to ways of composing smaller networks into larger ones, could be defined in terms of limits in \( \text{Auto} \). In particular, the operation of "feeding back" outputs to inputs was defined in terms of equalizers. We also showed that dataflow networks could be modeled more abstractly as spans in a category of \( \text{EvDom} \) of "conflict event domains," and that this model is related to the more concrete automaton model by a coreflection. Consequently, operations defined in terms of limits are preserved in the passage from the more concrete model to the more abstract version.

At the end of the previous paper, we noted several interesting properties, valid in the domain-theoretic model, of spans corresponding to dataflow processes, and we raised the question of what might be the correct categorical characterization of the "dataflow-like" spans. A proper answer to this question would be prerequisite to the construction of a fully categorical theory of dataflow networks. In the present paper, we obtain a characterization of dataflow-like spans as split right fibrations, either in a 2-category of automata, or in a 2-category of domains. The fact that essentially the same characterization holds in both cases lends credence to the idea that it is in fact the correct categorical notion. Further support comes from an intuitive interpretation of the definition of fibration. Fibrations in a 2-category are defined to be the algebras of a certain "doctrine," or 2-monad. In the present situation, this doctrine corresponds to the construction "compose with an input buffer." Thus, the dataflow-like spans are those spans that are algebras of the input buffering doctrine.

The theory of fibrations was first developed in terms of concrete constructions on categories [3]. Then, Street [14, 15], building on work of Gray [2], showed that this theory has a bicategorical formulation, which can be applied not only to the 2-category \( \text{Cat} \), but to any bicategory with sufficient completeness properties. Here, we examine how the theory applies to the category \( \text{Auto} \) of automata and the category \( \text{EvOrd} \) of "conflict event orderings," which is equivalent to the category \( \text{EvDom} \) of our previous paper. These categories have 2-categorical structure we have not exploited until now. As a technical matter, the 2-categories \( \text{Auto} \) and \( \text{EvOrd} \) do not quite have the necessary completeness properties (existence of comma objects and certain 2-pullbacks), so our results are complicated somewhat by the necessity of enlarging them to 2-categories \( \text{AutoWk} \), of "automata and weak morphisms," and \( \text{EvOrdWk} \), of "conflict event orderings and sup-preserving maps." We determine the structure of the split right fibrations in each of the