Studying Flexible Manufacturing Systems via Deterministic Control Problems

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Abstract

We deduce control equations governed by ordinary differential equations that determine the moments of the inventory levels in a flow control model of a flexible manufacturing system. Then we discuss how to use these control equations to study the expected inventory level set and the optimal inventory level control problem with quadratic cost functions.

1 Introduction

Managing failure prone flexible manufacturing systems is an important and challenging problem. An important aspect of the problem is to manage the inventory levels of various products of such a system. Since the machines in a flexible manufacturing system are subject to random failures the inventory levels are random variables at a given moment. The framework of flow control has been very successful in representing the random inventory levels. We will adopt the basic flow control model described in the recent books by Gershwin [6] and Sethi and Zhang [10]. There are two different routes one can take in studying inventory level management problems: treat the problem as a stochastic control problem or use the distribution functions of the inventory levels to model the problem as a deterministic control problem. There is much discussion on stochastic control models. In contrast deterministic control models are barely used. The major reason seems to be that the deterministic model derived by using the distribution functions of the inventory levels is, in general, a control system involving partial differential equations with controls that are coupled with the partial derivatives of the state variables. Such a control system is very difficult to handle both theoretically and computationally.

The main purpose of this note is to show that for some flexible manufacturing system management problems we can deduce deterministic control models that involve only ordinary differential equations. Since there are abundant results on the theory and computation of control systems governed by ordinary differential equations we will have more tools in our hands to study flexible manufacturing systems.

Our main observation is that the moments of the inventory levels can be determined by a control system governed by ordinary differential equations. Therefore, we can derive a control model involving only ordinary differential equations whenever our major concern is the moments of those random variables. Two important problems that fall in this category are the estimate of the expected inventory set and the optimal control of the inventory levels with a quadratic cost function. In Section 2 we state the general flow control model for a flexible manufacturing system and discuss the relations that determines the first and second moments of the inventory levels. Then we discuss how to apply these relations to the two concrete problems mentioned above in Section 3 and Section 4 respectively. In principle our method also applies to problems that involve higher moments of the inventory levels.

2 Flow Control Model for Flexible Manufacturing Systems and Moment Equations

We consider a general flexible manufacturing system that produces P part types on M machines. We assume that each machine in this system has two states: operational denoted by 1 and breakdown denoted by 0. Denote the state of the mth machine by s_m. Then $(s_1, ..., s_M)$ represents the state of the whole system. Let S be the finite set of system states, i.e., $S = \{(s_1, ..., s_M) : s_m = 0 \text{ or } 1, m = 1, ..., M\}$. Assume that for each state $s \in S$ the system has a corresponding production capacity set of various parts combinations that the system can produce in one time unit denoted by $C(s)$. In this model the M machines are treated as one 'block'. The set $C(s)$ describes the capacity of the