Recently there has been an increasing interest in studying the symmetry principles involved in classical mechanics. This interest is motivated by the identity of symmetry groups operating in classical and quantum mechanics. Indeed, there is the feeling that this common symmetry structure should be the guide for having a complete quantization procedure ("geometric quantization"). As a consequence, a fundamental problem arises: that of finding the symmetry group associated with a given mechanical system.

In this work we examine some features of this problem by means of the similarity analysis of a single particle's motion in Newtonian mechanics. In similarity analysis one usually tries to determine the general form of the differential equations (of some required order) which admit a given group as a symmetry group. Here we tackle the converse (and more interesting) problem of finding the Lie group symmetries of any given inhomogeneous ordinary differential equation of the second order: $\ddot{x} + f_2(t)\dot{x} + f_1(t)x = f_0(t)$. To this effect, we consider the symmetries generated by the infinitesimal point transformations: $t' = t + \varepsilon n(t,x)$, and $x' = x + \varepsilon \theta(t,x)$, as usual. Thus, we obtain the generators in the following forms:

$$n(t,x) = \phi_1(t)x + \phi_2(t)$$

$$\theta(t,x) = (\phi_1(t) - f_2(t)\phi_4(t))x^2 + \phi_3(t)x + \phi_4(t)$$

where $\phi_1, \ldots, \phi_4$ are functions of $t$, which are determined from a system of linear homogeneous differential equations; i.e.,

$$\ddot{\phi}_1 - f_2\dot{\phi}_1 + (f_1 - f_2^2)\phi_1 = 0$$

$$\ddot{\phi}_2 + (4f_1 - f_2^2 - 2f_2^2)\phi_2 + (2\dot{f}_1 - f_2\dot{f}_2 - \ddot{f}_2)\phi_2 = (f_0f_2 - \dot{f}_0)\phi_1 - 3f_0\phi_1$$

$$2\ddot{\phi}_3 = 3f_0\phi_1 + \ddot{\phi}_2 - f_2\ddot{\phi}_2 - \ddot{f}_2\phi_2$$

$$\ddot{\phi}_4 + f_2\ddot{\phi}_4 + f_1\phi_4 = 2f_0\phi_2 + f_0\phi_2 - f_0\phi_3$$

wherefrom,
\[ \eta(t,x) = q^a(\phi_{1.a}(t)x + \phi_{2.a}(t)) , \]
\[ \theta(t,x) = q^a((\phi_{1.a}(t) - f_2(t)\phi_{1.a}(t))x^2 + \phi_{3.a}(t)x + \phi_{4.a}(t)) . \]

The constants of integration \( q^a, a=1, \ldots, 8 \), behave as a set of eight essential parameters of the Lie group. Interesting enough, we have already sufficient information to formally obtain the associated Lie algebra. Indeed, we easily get the expressions

\[
 f_{ab}^c \theta_c = [\eta_a, \eta_{bt}] + [\theta_a, \theta_{bt}] ,
\]

where the \( f_{ab}^c \) denote the structure constants. (The antisymmetrization corresponds to the indices "a" and "b" only.) These are identities which hold for all \( t \); thus let us consider them at \( t=0 \). So, in order to represent the algebra, we adopt the following parametrization:

\[
 q^1 = \eta(0,0) , \quad q^2 = \theta(0,0) , \quad q^3 = \eta_x(0,0) , \quad q^4 = \theta_x(0,0) ,
\]

\[
 q^5 = \eta_x(0,0) , \quad q^6 = \theta_x(0,0) , \quad q^7 = \frac{1}{2}\eta_{xx}(0,0) , \quad q^8 = \frac{1}{2}\theta_{xx}(0,0) .
\]

Therefore, upon substituting from these initial data into the equations above, we obtain the following set of non-zeroth structure constants of the Lie algebra associated with the linear differential equation \( \dot{x} + f_2(t)x + f_1(t)x = f_0(t) \):

\[
 f_{13}^1 = 1 , \quad f_{15}^1 = -1 ; \quad f_{16}^2 = 1 , \quad f_{24}^2 = 1 ;
\]

\[
 f_{17}^3 = 2 , \quad f_{25}^3 = f_2(0) , \quad f_{28}^3 = 1 , \quad f_{56}^3 = -1 ;
\]

\[
 f_{13}^4 = \frac{1}{2}f_2(0) , \quad f_{15}^4 = \frac{3}{2}f_0(0) , \quad f_{17}^4 = 1 , \quad f_{28}^4 = 2 , \quad f_{56}^4 = 1 ;
\]

\[
 f_{15}^5 = f_2(0) , \quad f_{18}^5 = 1 , \quad f_{35}^5 = -1 , \quad f_{49}^5 = 1 ;
\]

\[
 f_{12}^6 = -f_1(0) + \frac{1}{2}f_2(0) , \quad f_{13}^6 = 2f_0(0) , \quad f_{14}^6 = -f_0(0) , \quad f_{16}^6 = -f_2(0) ,
\]

\[
 f_{23}^6 = \frac{1}{2}f_2(0) , \quad f_{25}^6 = \frac{3}{2}f_0(0) , \quad f_{27}^6 = 1 , \quad f_{36}^6 = 1 , \quad f_{46}^6 = -1 ;
\]

\[
 f_{13}^7 = -2f_1(0) + \frac{1}{2}f_2(0) , \quad f_{15}^7 = -f_0(0)f_2(0) , \quad f_{16}^7 = -\frac{3}{2}f_0(0) ,
\]

\[
 f_{25}^7 = -f_1(0) + \frac{1}{2}f_2(0) + \frac{1}{2}f_0^2(0) , \quad f_{28}^7 = \frac{3}{2}f_2(0) , \quad f_{35}^7 = f_0(0) ,
\]

\[
 f_{37}^8 = 1 , \quad f_{45}^7 = -\frac{1}{2}f_0(0) , \quad f_{56}^7 = -\frac{3}{2}f_2(0) , \quad f_{68}^7 = 1 ;
\]

\[
 f_{15}^8 = -f_1(0) + \frac{1}{2}f_2(0) , \quad f_{35}^8 = \frac{1}{2}f_2(0) , \quad f_{48}^8 = 1 , \quad f_{57}^8 = 1 .
\]