Abstract

In this paper we discuss a collection of geometric location problems in the plane and their associated time complexity. These problems can be formulated as optimization problems. However, geometric properties are exploited to obtain efficient solutions. Among the problems considered are minimum enclosing circle, largest empty circle, fixed circle placement, and their variations. Most of the algorithms mentioned are asymptotically optimal to within a constant factor under the algebraic computation tree model. Finally, a geometric competitive location problem is discussed and some open problems suggested.

1. Introduction

Recently the newly emerged field, known as computational geometry [30,31], has received a great deal of attention due to its applicability to other various disciplines. It deals with geometric problems within the framework of design and analysis of algorithms. A recent survey on this subject can be found in [19]. For many problems geometric properties can be utilized to obtain a faster solution. For instance, the Euclidean minimum spanning tree of a set of n points in the plane can be constructed in $O(n \log n)$ time, whereas the graph-theoretic counterpart requires at least $\Omega(n^2)$ time, which is the time needed to construct the graph (a complete graph

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with the points as vertices and $O(n^2)$ edges with edge weight equal to the Euclidean distance between two endpoints of the edge). Of course, for some problems, e.g., Euclidean traveling salesman problem (ETSP), geometric properties do not help. Both the graph-theoretic and geometric versions of ETSP are known to be NP-complete [12,29]. Thus, for theoreticians, the problem of whether geometric properties of a given problem can be useful to reduce the time complexity is of great interest. Along this line researchers are interested to know, on the one hand, how "difficult" a given geometric problem is and find an efficient solution to the problem, on the other. If the two bounds (lower and upper bounds) for the problem are identical, i.e., have the same order of magnitude, they will define the intrinsic complexity of the problem. In this paper, we shall address a collection of geometric location problems and their complexities. For most of the problems mentioned, the intrinsic complexity is $O(n \log n)$, where $n$ denotes the size of the problem. We assume the following computational model. Basically, we have a random access machine (RAM) with real arithmetics [1]. That is, any real number can be represented in one word, and can be accessed in constant time. Unit cost is assumed for all basic arithmetic operations. Certain primitives, e.g., computing the distance between two points, intersection of two straight lines, etc. are assumed to take constant time (independent of the input size). Trigonometric functions of the input are avoided, if possible. More precisely we use the so called algebraic computation tree model of Ben-Or [3]. In what follows we shall discuss a few geometric optimization problems that arise in operations research. The interested reader is referred to the original papers pertaining to a specific problem for more details.

2. Smallest Enclosing Circle Problem

Given a set $S$ of $n$ points, $p_1, p_2, \ldots, p_n$, in the plane we want to find a circle of smallest area that encloses all the points. This problem is also known as the 1-(point-) center problem in location theory [11], and is usually formulated as a minimax problem, i.e.,

Given $S = \{p_1, p_2, \ldots, p_n\}$, $p_i = (x_i, y_i)$, $i = 1, 2, \ldots, n$,
find $C = (x, y)$ such that $\max_{i} d(C, p_i)$ is minimized,
where $d(a,b)$ is the Euclidean distance between points $a$ and $b$. 
