Deciding Reachability for Planar Multi-polynomial Systems *

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Abstract. In this paper we investigate the decidability of the reachability problem for planar non-linear hybrid systems. A planar hybrid system has the property that its state space corresponds to the standard Euclidean plane, which is partitioned into a finite number of (polyhedral) regions. To each of these regions is assigned some vector field which governs the dynamical behaviour of the system within this region. We prove the decidability of point to point and region to region reachability problems for planar hybrid systems for the case when trajectories within the regions can be described by polynomials of arbitrary degree.

1 Introduction

During recent years intensive research has been devoted to the problem of automated analysis of various classes of hybrid systems (HS). The difficulty of this problem is due to the presence of a continuos projection of the system state space (every system state of a HS typically consists of control location, which is chosen from some finite domain, and the value vector for some continuous variables), this usually makes the system state space (wildly) infinite. However, there has already been much progress in the area, starting from the region graph based methods for Timed Automata [2, 3, 6], and leading to recent more general results and systematic investigations on what is decidable about hybrid systems (see, for instance, [1, 9, 8]).

Still, most of these results are concerned with the analysis automation for linear hybrid systems, where the continuous variables are allowed to change the value during the course of time at some fixed rate (or the value change can be non-deterministic, with any rate from a certain fixed interval).

It can be noted, however, that for the full class of linear HS even the simplest verification problems are undecidable (see, for instance [6, 5, 1]), so any decidability result in this area is bound to indentify a certain subclass of systems to which it does apply.

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On the other hand, the behaviour of practical hybrid systems is most often governed by some non-linear laws. Therefore it is natural to ask, whether there are natural classes of non-linear HS, which do admit automated verification. An important study of this problem is already [7], where the possibility of verification on nonlinear hybrid systems via the reduction to linear clock and rate hybrid automata is discussed, and corresponding at least sound verification methods are presented.

In this paper we investigate the decidability of the reachability problem for planar non-linear hybrid systems. A planar hybrid system has the property that its state space corresponds to the standard Euclid plane, which is partitioned into a finite number of (polyhedral) regions. To each of these regions is assigned some vector field which governs the dynamic behaviour of the system within this region. We prove the decidability of point to point, edge to edge and region to region reachability problems for planar hybrid systems for the case when trajectories within the regions can be described by polynomials of arbitrary degree.

Our results are a generalization of those of [10], where the subcase of our problem with the vector fields within the regions being constant (the so-called multi-linear model) was considered. We are able to reuse also a part of the proof from [10] to show that every infinite trajectory of the system either intersects only with a finite number of the region boundaries, or starting from some point will repeatingly intersect certain fixed sequence of region boundaries (in fact this result holds even for much wider classes of systems, its demonstration relies essentially on the fact that trajectories within the regions do not intersect).

The main problem to be dealt with in our “multi-polynomial” case essentially consists in showing the decidability of the “abandonment” of an edge (region boundary, or some its part): given some repeating sequence of edges intersected by a trajectory, decide, whether this repetition will last forever, or after some finite number of edge-to-edge steps some other edge intersection sequence will appear.

This problem is solved, in essence, by explicating the polynomial dependencies of future region border intersection points from the previous ones on the trajectories, and characterizing the “limit points” of the intersection point sequences in the terms of fixed points and roots of appropriate polynomials.

Our results can be viewed as showing the non-essentiality of the linearity requirement for the decidability results in the setting of [10]. However, as it has been shown in [4], the 2-dimensionality requirement is essential for the decidability of the reachability even for the case of multi-linear systems (in [4] a construction modeling Turing machines by 3-dimensional multi-linear systems is presented, thus proving the undecidability of any nontrivial verification problem for that class of systems).

The organization of the rest of the paper is, as follows. In the next section we give main definitions and notation used throughout the paper. Section 3 reminds already known results about general planar hybrid systems. Section 4 contains our results about decidability results for multi-polynomial systems.