Stratified Default Theories

Pawel Cholewinski

Department of Computer Science, University of Kentucky, Lexington, KY 40506

Abstract. Default logic is a nonstandard formal system, especially suitable for knowledge representation and commonsense reasoning. In this paper we study a class of propositional default theories for which computation of extensions simplifies. We introduce the notions of stratification and strong stratification. We investigate properties of stratified default theories. We show how to determine whether a given default theory is stratified or strongly stratified and how to find the finest partition into strata. We present algorithms for computing extensions for stratified default theories and analyze their complexity.

1 Introduction

Nonmonotonic reasoning systems allow us to formalize reasoning with incomplete information and other forms of commonsense reasoning. They model a behavior of an agent who constructs its knowledge (or belief) sets by reasoning not only from what is known to him but also from what is possible or consistent to assume. Default logic of Reiter [13] proved to be one of the most successful nonmonotonic reasoning systems. Its applications range from databases to expert systems and planners.

Default logic can be regarded as a proof system obtained from propositional logic by adding nonstandard inference rules called defaults. The difference between a default and a standard inference rule is the presence of two kinds of premises – prerequisites and justifications. The role of prerequisites of defaults is the same as the role of premises in standard inference rules – they have to be proven before we can apply a default. But prerequisites of the second type – justifications – do not need a proof. They need to be shown possible in order for a default to be applicable. Any default theory represents a family (possibly empty) of all possible knowledge (belief) sets, which are called extensions. Due to the fact that justifications do not need proofs but only have to be possible, default reasoning is defeasible. That is, some inferences may become invalid when more facts become known.

Default reasoning problems have high computational complexity. For instance, the problem of deciding if a given formula \( \varphi \) belongs to at least one extension of a finite default theory is \( \Sigma^P_2 \)-complete and deciding whether \( \varphi \) belongs to all extensions is \( \Pi^P_2 \)-complete [8], [12]. In fact, these complexity results hold even if we restrict to normal default theories with all defaults prerequisite free. Even if we restrict to the class of disjunctive-free theories these problems are still NP-hard [9].
To improve efficiency of reasoning with default logic it is worth to check if a given theory belongs to a subclass for which reasoning can be performed faster. A common technique is to stratify a theory. Stratification consists of partitioning a given theory into a sequence of smaller theories for which extensions can be computed faster. This approach was widely studied in the cases of logic programming and autoepistemic logic ([1], [2], [3], [7], [14]) Some results of applying this approach to default logic were obtained by Etherington [6] and Kautz and Selman [9]. Their results were based on restricting the syntactic forms of formulas used in default theories.

In [5], we studied the class of seminormal default theories. We did not impose there any syntactic restrictions on formulas appearing in defaults, but more restrictive conditions on dependencies between defaults were required. We showed that every strongly stratified seminormal default theory has an extension, and that each ordering of defaults which agrees with a strong stratification generates an extension. Conversely, each extension for such a theory is generated by some ordering which agrees with stratification. Moreover, extensions of a stratified seminormal default theory can be found by considering the theory stratum by stratum.

In this paper we study general default theories. That is, no syntactic restrictions on the form of defaults are required. We generalize the concept of stratification for seminormal default theories introduced in [5] to the case of arbitrary default theories. Under a very weak additional assumption (each default must have at least one justification) all results from [5] extend to this case. In particular it is possible to find extensions by considering the theory stratum by stratum which often yields significant speedups in reasoning algorithms.

We also introduce a concept of an extension tree. For a default theory \((D, W)\) with \(D\) stratified into \(D_1, \ldots, D_k\), an extension tree is a tree with theory \(W\) in its root and nodes on level \(i\) being all the extensions of the default theory \((D_1 \cup \ldots \cup D_i, W)\). We use extension trees to construct algorithms for computing extensions and query processing for default theories. Finally we show that checking if a given default theory is stratified and computing the finest partition into strata is easy and can be done in polynomial time.

### 2 Preliminaries

Throughout this paper we assume that every default has at least one justification. We consider formulas built over a given propositional language \(\mathcal{L}\). For any propositional formula \(\varphi\) by \(\text{Var}(\varphi)\) we denote the set of all propositional variables which appear in \(\varphi\). For every justification \(\beta_i\) of a default

\[ d = \frac{\alpha : \beta_1, \ldots, \beta_k}{\gamma} \]

we define the set of its conflict variables, denoted \(\text{Var}^*(\beta_i)\), as follows:

1. if \(\beta_i = \gamma\) (normal justification) then \(\text{Var}^*(\beta_i) = \emptyset\);