A HOMOMORPHISM CONCEPT FOR $\omega$-REGULARITY

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ABSTRACT. The Myhill-Nerode Theorem (that for any regular language, there is a canonical recognizing device) is of paramount importance for the computational handling of many formalisms about finite words.

For infinite words, no prior concept of homomorphism or structural comparison seems to have generalized the Myhill-Nerode Theorem in the sense that the concept is both language preserving and in a natural correspondence to automata.

In this paper, we propose such a concept based on Families of Right Congruences [3], which we view as a recognizing structures.

We also establish an exponential lower and upper bound on the change in size when a representation is reduced to its canonical form.

1. Overview

An important and only partially solved problem in the theory of $\omega$-regular languages is whether representations can be minimized. For usual regular languages, deterministic finite-state automata (DFAs) are recognizing structures that can be minimized easily in polynomial time by virtue of the Myhill-Nerode Theorem. Although $\omega$-regular languages enjoy some of the same properties as regular languages, see [6], the lack of similar minimization algorithms is a major impediment to building verification tools for concurrent programs.

The syntactic congruences of Arnold [2] provide canonical algebraic structures for $\omega$-regular languages. By themselves, these congruences provide no explicit acceptance criteria just as in the situation for a regular language: the canonical right congruence, whose classes are automata states, does not define a

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language—unless certain states are designated as being final. Similarly, Arnold’s congruences have only the ability to recognize, which is a property called saturation. Arnold’s congruences can be extended so that acceptance becomes explicit and thus a language preserving homomorphism concept arises. But, unlike the Myhill-Nerode Theorem, which is based on right congruences, canonicity in [2] is obtained for full congruences, which are usually exponentially bigger than one-sided congruences.

Maler and Staiger [3] focus on the canonical right congruence $\equiv_L$ on finite words of a language $L$ of infinite words. This congruence is defined by $x \equiv_L y$ if and only if for all infinite $\alpha$, $x \cdot \alpha \in L$ if and only if $y \cdot \alpha \in L$. (We use $x, y, u, v, w$ to denote finite words and $\alpha, \beta$ to denote infinite words). The concept of a Family of Right Congruences (FORC) suggested in [3] is there used to characterize $\omega$-regular languages that are accepted by their canonical right congruence $\equiv_L$ extended to a Muller automaton.

FORCs are also not language recognizing. But they do enjoy canonical properties with respect to saturation as we prove in this paper. Similarly, the right binoids of Wilke [8] are algebraic devices that characterize regular sets of finite and infinite words based on a saturation concept embedded in a notion of recognition by homomorphism.

Other algebraic approaches to $\omega$-regularity include the semigroup approach of [5] and the right congruences proposed in [4]. The latter congruences are defined in terms of language recognizing automata, but only some $\omega$-regular languages are recognizable by their canonical automaton defined by their language [7].

In this paper. In this paper we regard FORCS as language accepting devices rather than as the transition structures of underlying Muller automata. Then FORCS may be viewed as separating the characterization of the topological closure of the language from that of the dense part.

The closure corresponds to the canonical right congruence. The classes of this relation for which there is an infinite suffix that makes words in the class belong to $L$ describe the closure of $L$: an infinite word is in the closure if and only if all of its prefixes belong to these classes. The closure is also called a safety property in the theory of concurrent systems. A FORC represents the closure by what we here call a safety congruence, which is a refinement of the natural right congruence. (The results of [3] show under which conditions this safety congruence may be used with a Muller condition to accept languages that are not necessarily closed.)

The dense part of $L$ is described by a collection of right congruences, here called progress congruences, that specify the cyclic behavior any word eventually exhibits according to Ramsey’s Theorem about finite partitions of the natural numbers. Thus it is natural to view these congruences as an algebraic formalization of progress towards the dense part, known as a liveness property in concurrency [1].

We show that a Myhill-Nerode Theorem exists that declares a unique minimum representation of an $\omega$-regular language under a structural comparison