Feasibility of Finite and Infinite Paths in Data Dependent Programs

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Abstract

This paper considers the feasibility of finite and infinite paths in programs in two simple programming languages. The language LBASE allows to express the dependencies of real time systems on integer data, the language LTIM can model quantitative timing constraints in r.t.s. specifications. It is proven that the problem of whether a given LBASE or LTIM program has an infinite feasible path (i.e. whether it can exhibit an infinite behaviour) is decidable. The possibilities to characterise the sets of all feasible finite and infinite paths in LBASE and LTIM programs are also discussed. The infinite feasible path existence problem is proven decidable also for the language LTIBA which has both the LBASE and LTIM modelling capabilities.

1 Introduction

Despite the simplicity and the decidability of all interesting algorithmical problems for Finite State machines (FSMs) the FSM model does not appear to solve completely the real time system (r.t.s.) correctness problem because of its insensitivity to various aspects of the system behaviour.

This paper considers two kinds of enrichments of the FSM model with a variable notion, we call them the programming languages LBASE and LTIM. The language LBASE contains means for modelling the dependence of the r.t.s. behaviour on integer-valued parameters (the decision in LBASE programs can be made by comparing two variables on being less or greater, arithmetical transformation of data in LBASE programs is not allowed), it has been already widely studied (see, for example [BBK77, ABBCK91]) with respect to the statement reachability and automatic complete test set generation problems (test set completeness criterion - $C_1$ - covering all feasible branches). What makes any automated reasoning about LBASE programs nontrivial is the infinity of the domain of program variable values, it leads also to the infinity of the "state space" for the most of the programs, so the standard FSM analysis methods, based on the state-space exploration, cannot be directly applied.

All the work done previously for the analysis of LBASE programs was oriented to the applications in non-real-time programs; the program's execution in the "white-box" approach was normally characterized by the feasibility of finite paths in it. With respect to the analysis of r.t.s. a number of important properties characterizing the infinite behaviour of the system arise, for instance, can a given real time system exhibit an infinite behaviour.

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at all, or it always is forced to terminate after a finite number of execution steps; can such
an infinite behavior infinitely many times visit some desirable states; is a given node in the
program graph reachable within an infinite behaviour, etc.? One of the main objectives
of this paper is to show that all the abovementioned infinite path feasibility problems for
LBASE programs are decidable. We discuss also some possibilities (or, rather, impossibil-
ities) to give some characteristics of the set of all infinite feasible paths for given LBASE
program, notice that an elegant such characteristic exists in the case of finite feasible paths.

Another programming language under discussion in this paper is LTIM, the programs in
it are allowed to have a finite number of rational-valued variables, as well as a nondecrasing
counter with some assignment operations in both directions between it and ordinary vari-
ables allowed. The main reason for considering this language when studying the real time
systems is its ability to express the quantitative timing constraints on the system behaviours
(such as "the event b occurs not later than 5 seconds after the event a"). The techniques,
developed in [ABBCK91, Cer91a] for the reachability analysis of LTIM programs can be
widely used in the analysis of timing constraints in various timed specification formalisms,
including Time Petri Nets [MF76, BM83, GMMP89], timed process algebras (such as Timed
CCS [Wan90]) and, as a more practical example, the C.C.I.T.T. specification language SDL
[CC88]. Actually, many of the high level timed specification formalisms have developed their
own analysis techniques, one can see [BM83, ACD90, AD90] for a related work. The novelty
of our approach can be seen in the modelling of the passage of time by the means of stan-
dard data structures (some kind of counter) and treating the time moment information as
data for the programs. The language LTIM has also some possibilities to describe slightly
more general time dependencies than the most of other formalisms (Time Petri Nets, Timed
Graphs [ACD90], Parallel Timer Processes [Cer91b, Cer92a]) with decidable reachability
problem (see [ABBCK91] for some applications w.r.t. SDL).

In this paper this approach is continued by studying the feasibility of infinite paths in
LTIM programs. It is proved here that the set of all infinite feasible paths in each LTIM
program can be effectively characterised by the set of all fair paths in some FSM with a
fairness condition (some nodes visited infinitely often), so also proving that the problem of
the existence of an infinite feasible path in a given LTIM program is decidable.

When viewing LTIM programs just as programs with some kind of counter, it is inter-
esting to point out that the statement reachability problem for them becomes undecidable
if we consider the defined language operations over integer-valued variables, instead of the
rational-valued ones in the original definition. This result gives a new class of programs
with a counter having the reachability problem undecidable.

Having modelled the timing constraints in r.t.s. by means of a simple programming
language we come up with a formalism for specifying both the data- and time-dependent
behavior of r.t.s. We introduce a programming language LTIBA containing both the facili-
ties of LBASE and LTIM, the main result in the paper states that the problem of whether
a given LTIBA program has an infinite feasible path (i.e. whether it can exhibit an infi-
nite behaviour) is decidable. As a slight generalization of this result one obtains also the
decidability of the infinite fair path existence problem in given LTIBA program.

Finally, let us note that the vertex reachability and (finite or infinite) path feasibility
problems are not the only problems which arise in verification of real time systems. For time-
dependent programs there are also model checking (see [ACD90]) and various equivalence
problems studied. In [AD90] the trace equivalence problem for Timed Büchi automata is
proven undecidable, [Cer91b, Cer92a] shows the decidability of bisimulation equivalence for
Parallel Timer Processes. The equivalence problems for data-dependent real time programs
seem to be much less investigated, perhaps they are also harder.