I. Introduction.

Recently, there has been increased interest in non-first-normal-form relation structures. Allowing set structures to be entries in a tuple of a relation is both of theoretical interest (cf. [JS, FT, T]) and of potential practical value (cf. [BRS, AB]).

So far, the best-analyzed operation to restructure relations is the NEST operator, which was defined in [JS] and independently studied in [T]. Informally, nesting on a set of attributes Y collects into sets those subtuples over Y which agree on all values outside of the nesting attributes Y.

The weak multivalued dependency was introduced in [JS] as a characterization of when two NEST operations would commute. Weak mvd's were analyzed in [FvG], where a complete axiom system was presented. It was also shown in [FvG] that weak mvd's interact very little with each other so that, given $n \geq 2$ pairwise disjoint sets of attributes, any two of the sets may have permutable NEST operations independently of any other (unordered) pair of sets of attributes. Furthermore, the case where nesting is fully permutable over $n$ sets of attributes was characterized in terms of a specific family of weak mvd's.

In this paper we study the effects of multiple nesting by looking more closely at the horizontal decomposition induced on a relation by nesting. Informally, one performs NEST operations on a relation $r$, creating a relation $r^*$ consisting of tuples with set-valued entries in some positions, then considers those tuples of $r$ which contribute to a tuple of $r^*$. The result is a partitioning $B$ of the original relation into blocks, each associated with a tuple of the nested structure $r^*$. We show that the relationship between the sets of values appearing in different blocks of $B$ correspond closely to the satisfaction of certain families of mvd's or certain families of weak mvd's by $r$.

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In particular, whether nesting permutes fully can be characterized by analyzing the blocks of $B$.

2. Preliminaries.

Throughout this paper the following notations and definitions will be used. $U = \{A_1, \ldots, A_m\}$ is a finite set of attributes, called the universe. $\text{DOM}(A)$ is the set of domain values associated with attribute $A$. A tuple $t$ over $U$ is a mapping from $U$ into $\bigcup_{A \in U} \text{DOM}(A)$ such that for each $A \in U$, $t(A) \in \text{DOM}(A)$. The $X$-value of tuple $t$, where $X \subseteq U$, is the restriction of this mapping to $X$ and is denoted $t[X]$. A relation $r$ over $U$ is a set of tuples over $U$. The projection of a relation $r$ on $X$ is the set $\{ t[X] | t \in r \}$ and is denoted $\Pi_X(r)$. Let $r$ be a relation over $R$ and $s$ a relation over $S$, then the join of $r$ and $s$ is the relation $r \Join_X s$ over $R \cup S$ consisting of all tuples $t$ such that $t[R] \in r$ and $t[S] \in s$.

A relation $r$ satisfies the multivalued dependency (MVD) $X \rightarrow Y$ if and only if whenever $r$ contains tuples $t_1$, $t_2$ such that

\[
\begin{array}{c}
X & Y & Z \\
\hline
x & y & z \\
t_1 \\
x & y' & z' \\
t_2 \\
\end{array}
\]

then $r$ contains a tuple $t_3$ such that

\[
\begin{array}{c}
X & Y & Z \\
\hline
x & y & z \\
t_3 \\
\end{array}
\]

We also say that $r$ satisfies the template dependency $(t_1, t_2)/t_3$.

A relation $r$ satisfies the weak multivalued dependency (WMVD) $X \rightarrow^w Y$ if and only if whenever $r$ contains tuples $t_1$, $t_2$, $t_3$ such that

\[
\begin{array}{c}
X & Y & Z \\
\hline
x & y & z \\
t_1 \\
x & y' & z' \\
t_2 \\
\end{array}
\]

\[
\begin{array}{c}
X & Y & Z \\
\hline
x & y' & z \\
t_3 \\
\end{array}
\]

then $r$ contains a tuple $t_4$ such that

\[
\begin{array}{c}
X & Y & Z \\
\hline
x & y' & z' \\
t_4 \\
\end{array}
\]

We also say that $r$ satisfies the template dependency $(t_1, t_2, t_3)/t_4$.

Let $R_1, R_2, \ldots, R_m$ be subsets of $U$. A relation $r$ satisfies the join dependency $|X|(R_1, \ldots, R_m)$ if and only if

\[
r = \Pi_{R_1}(r) \Join_{X \ldots X} \Pi_{R_m}(r).
\]