Abstract: In this paper we propose some syntactical criteria on algebraic specifications that ensure completeness of narrowing strategies. We then prove a theorem relating narrowing and reduction relations. The completeness of narrowing strategies is proved and conditions for the computation of a "minimal" ground complete set of E-unifiers are given; as well as an algorithm transforming specifications satisfying Huet and Hullot's principle of definition, into specifications fulfilling the proposed criteria.

1 Introduction

During the few last years many investigations attempting to unify relational and functional languages have been made [2,3,5,7,8,9,14,20..]. Most of these proposals or implementations are based on E-unification [6,13]. The aim of this paper is to discuss completeness of strategies that implement E-unification using the narrowing relation [22]. In [6], Fay was the first to propose an algorithm for solving equations in a given equational theory which defines a canonical term rewriting system. The implementation of the proposed algorithm is too costly in time and space. Hullot [13] and recently Réty [21] improved the former algorithm by reducing the search space using basic narrowing. In [4] another algorithm close to the two preceding ones inspired from the similarity between narrowing and resolution[1] has been suggested. However, in spite of the proposed improvements, the set of E-unifiers computed in each case still contains redundancies in general. On the other hand, some authors investigate more pragmatic strategies that consist in performing narrowing derivations at only one occurrence per equation. Then the search space becomes smaller than the previous ones. But in this case we are rather concerned with ground E-unifiers. As an example of such strategies we quote the so-called "innermost strategy" which has been proved complete, up to some conditions on the equational presentation, by Fribourg [7]. Unfortunately, such strategies are not always complete, for instance the outer-most strategy is not complete as shown in example 1. In a recent work [19], Padawitz gave some sufficient conditions on goals (equations in this paper) that he called "uniformity", which ensure the completeness of any narrowing occurrence selection strategy, and he proposed a characterisation of "uniformity" only for
the leftmost-outermost strategy. In this paper we propose similar results and give some syntactical criteria on specifications that we may obtain by automatic transformation of specifications. Those conditions allow us to prove the completeness of an arbitrary narrowing occurrence selection strategy. We also give sufficient conditions in order to compute a "minimal" ground complete set of E-unifiers.

In the next section, we briefly recall some relevant definitions and fix notations for the understanding of the paper. We introduce then, in the third section, the syntactical conditions on specifications. In section four we prove a new lemma connecting reduction and narrowing relations. Section five aims to prove completeness of an arbitrary narrowing occurrence selection strategy. In section six, we give a procedure of transformation of specifications and prove its correctness.

2 Preliminaries

We assume the reader is familiar with the basic notions of many-sorted algebras and term rewriting systems. The missing definitions could be found for instance in [12,17]. Notations used throughout the paper are consistent with [12,17].

Definition 1: An algebraic signature \( \Sigma \) is a pair \((S, \Omega)\) with \(S\) being a set (of sorts) and \(\Omega\) a family of sets (of operators) indexed by \(S^*\times S\). \(f\) in \(\Omega_{u,s}\), denoted by \(f:u\rightarrow s\), has as **arity** \(u\), **sort** \(s\) and \(\text{rank } u,s\).

Definition 2: Let \(\Sigma=(S, \Omega)\) be a signature and \(X\) an \(S\)-indexed set (of variables) such that \(\Omega \cap X = \emptyset\). We denote by \(T(\Sigma,X)\) the free \(\Sigma\)-algebra over \(X\) (also called term algebra). Its carriers are denoted by \(T(F_\Sigma,X)_S\). When \(X\) is empty, \(T(\Sigma)\) denotes the initial (up to isomorphism) \(\Sigma\)-algebra. Let \(t\) be a term in \(T(\Sigma,X)_S\), by \(\text{vars}(t)\) we denote the variables occurring in \(t\). If \(\text{vars}(t)\) is empty, \(t\) is said to be **ground**. A term \(t\) is **linear** iff every variable \(x\) in \(\text{vars}(t)\) occurs only once in \(t\). In order to denote subterms, we shall use occurrences (sequences of naturals standing for addresses of subterms). \(O(t)\) denotes the set of **occurrences** of \(t\); it is defined as follows:

1. \(\epsilon \in O(t)\), \(\epsilon\) is the empty sequence.
2. \(u \in O(t_i) \Rightarrow i.u \in O(f(t_1, \ldots, t_i, \ldots, t_n)), \forall i \in \{1, \ldots, n\}, f \in \Omega\).

\(O^*(t)\) denotes the subset of non variable occurrences of \(O(t)\). \(O(t)\) is partially ordered by the prefix ordering: \(u \leq v\) iff there exists \(w\) such that \(u.w = v\).

We write \(t/u\) to denote the subterm of \(t\) at occurrence \(u\). This is defined as follows:

1. \(t/\epsilon = t\)
2. \(f(t_1, \ldots, t_i, \ldots, t_n) / i.u = t_i/u\)

We denote by \(t[u<- t']\) the term obtained by replacing \(t/u\) by \(t'\) in \(t\).