1 Introduction

Natural deduction was introduced by Gentzen [7] in 1934 as a formalization of rigorous human reasoning. It turned out to be especially suitable for extracting programs from proofs. This program interpretation (Curry-Howard isomorphism of natural deduction and lambda-calculus) gave rise to the slogan PROOFS=PROGRAMS. Close connections with formulations suitable for immediate proof search (Prawitz trasformation to sequent systems, [33] 1965, and Maslov transformation to resolution, [17] 1973) were discovered remarkably late.

This paper is a survey of several results in these directions. Some of the proofs are only sketched and will appear in other papers.

First we consider in the section 2 standard assignment of λ-terms to natural deductions in the (~:,→)-fragment of the intuitionistic propositional logic (Curry-Howard isomorphism). We compare two familiar methods for extraction of programs from proofs in the intuitionistic sequent calculus. One method going back to Gentzen [7] is local: every step in the proof is transformed into a fixed construct of λ-calculus (added to what is already constructed). It introduces cuts. The second method (cf. [33, 26, 32]) is global: it makes a substitution for some variables and does not introduce cuts. It turns out (Theorem 1, cf. [28]) that these transformations differ only in a series of non-iterated β-conversions: the second one is obtained from the first by a complete development [4] of (redundant) redexes introduced to make the transformation global. Viewed as intertranslations between sequent system LJ and system NJ of natural deduction, local methods use the direct derivability of the rules of one system in the other, and so are linear in the number of inferences, while global methods use cut-free admissibility of the rules, and are exponential in the worst case.

Natural deduction is used both for exact formalization of reasoning and as a tool for extraction of programs from proofs and normalization of proofs (deductions) which exactly corresponds under Curry-Howard isomorphism to computation by corresponding programs. In both of these situations standard θ-elimination rule presents well-known difficulties. The rule of existential instantiation ∃xA(x)/A(a), which was introduced with the intention to bring
formalization closer to human reasoning (the history is presented in [35, 33, 14]) allows to overcome some of these difficulties.

In the section 3 we describe a manageable formulation NJi of intuitionistic logic where restrictions on eigenvariables are made local and reduced to a minimum by a device from [31, sections 7,8], cf. also [24]. The only new syntactical objects are assumptions \(< \Gamma \rangle \exists x F[x/a]$, which cannot be combined into more complicated formulae, and are eventually interpreted as $F[x/a]$. System $Me$ of [14] is probably the closest to NJi in the literature. Main differences with $Me$ are the use of $\varepsilon$-terms there and a different mechanism of restricting assumptions. Detailed comparison of $Me$ with other systems is presented in [14]. We present also a rule for disjunction which can play the same role for $\forall$ that $\exists i$ plays for $\exists$. Only intuitionistic predicate logic is considered here, but extension to classical or higher order case is straightforward. A translation $*$ of NJi into NJ is defined and proved to preserve reductions (normalization steps) of proofs. This proves soundness and completeness of NJi and allows to derive strong normalization for NJi from strong normalization for NJ.

In the section 4 we describe an extension of Curry-Howard isomorphism CH [5, 11] to the $!$-free fragment ($\otimes, \&, \rightarrow, I$) of intuitionistic propositional linear logic. Proofs are presented in [29]. The basis is provided by the results of two previous papers: [24] contains a proof of a normal form theorem for the second order Intuitionistic Predicate Linear Logic, and [22] treated $\lambda$-terms for (what is now called) ($\otimes, \rightarrow, I$)-fragment of intuitionistic propositional linear logic. Some of the related papers including [3, 23, 36, 39, 41] are discussed in [24]. We obtain a unique normal form for natural deduction in ($\otimes, \&, \rightarrow, I$) fragment of the intuitionistic linear logic demonstrating a possibility to treat considerable fragment of linear logic in the framework of standard lambda calculus adding no new combinators. This possibility was realized in [22, 2] for the ($\rightarrow, \otimes I$)-fragment, but proofs in [22] are presented only for ($\rightarrow, I$)-fragment, and [2] is not easily accessible.

Section 5 illustrates connection between natural deduction and resolution for intuitionistic logic.

2 Translations of sequent calculus into natural deduction

In this section we consider $\&, \rightarrow$-fragment of intuitionistic logic.

Derivable objects of Gentzen's system $LJ$ are sequents

$$A_1, \ldots, A_n \Rightarrow D$$

(1)

In a term assignment assumptions $A_1, \ldots, A_n$ are assigned distinct variables $x_1, \ldots, x_n$ of types $A_1, \ldots, A_n$ and formula $D$ receives a term $u$. A sequent (1)