Subgoal Alternation in Model Elimination*

Ortrun Ibens and Reinhold Letz

Institut für Informatik
Technische Universität München
80290 München, Germany
{ibens, letz}@informatik.tu-muenchen.de
phone +49-(0)89-521097

Abstract. The order in which subgoals are selected and solved has a strong influence on the search space of model elimination procedures. A general principle is to prefer subgoals with few solutions over subgoals with many solutions. In this paper we show that the standard selection methods are not flexible enough to satisfy this principle. As a generalization of the standard paradigm the new method of subgoal alternation is presented and integrated into the theorem prover SETHEO. Among other advantages, subgoal alternation also provides more look-ahead information about the needed proof resources than the standard method; this information can be used for search pruning. The evaluation of the new technique on a large number of formulae shows a significant improvement in performance.

1 Introduction

The model elimination calculus [Lov78] can be viewed as a refined tableau method, namely the free-variable tableau system [Fit90] with the connection condition [LSBB92, LMG94]. The particular suitability of model elimination for automated deduction results from its goal-directedness, which is ensured by the connection condition. The same condition, however, is responsible for the main disadvantage of the calculus, its non-confluence. In order to find a proof in model elimination, one has to enumerate all possible deductions, in contrast to general tableaux, where it is sufficient to work on one tableau.

The search space of model elimination can be seen as a complex form of an and-or-tree in which tableaux are and-nodes (connecting the contained subgoals) and subgoals are or-nodes (connecting the possible unification partners). While, for any subgoal, all possible unification partners have to be tried in order to guarantee completeness, soundness requires to solve all subgoals. The order of subgoal selection strongly influences the size of the search space. This is because subgoals normally share variables and thus the solution substitutions of one subgoal have an influence on the solution substitutions of the other subgoals.

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A general least commitment paradigm is to prefer subgoals that produce fewer solutions.

This has been recognized, for example, in the logic programming community, and respective heuristics have been developed for subgoal selection. One common feature of all standard subgoal selection methods, however, is that, once a subgoal has been selected, one sticks to that preference for all its unification partners, which is also a basic search paradigm of Prolog. In this paper, we show that this paradigm has a fundamental search-theoretic weakness. It is demonstrated that one can profit from changing the subgoal preference dynamically, depending on the remaining unification partners in the or-nodes. We develop a new method of subgoal alternation which is more flexible than the standard approach and can better satisfy the aforementioned fewest-solution principle.

The integration of subgoal alternation into model elimination also benefits from a number of synergetic effects with other search properties. On the one hand, one can profit from look-ahead information concerning the minimal number of inferences needed for closing a tableau. On the other hand, local failure caching [LMG94] is supported by subgoal alternation. The new method has been implemented within the model elimination theorem prover SETHEO. In combination with other techniques contained in the system, a significant performance gain could be obtained, particularly for the special equality handling in SETHEO. The use of subgoal alternation is one of the main reasons why SETHEO was a winner in the first world-wide theorem prover competition at the Conference on Automated Deduction 1996 [MIL+97].

The paper is organized as follows. In Section 2 we give a short introduction to the underlying proof procedure. Section 3 explains the general selection principles for subgoal selection. In Section 4 we illustrate the weakness of standard subgoal selection and present our new approach. In Section 5 the integration of the subgoal selection strategies in SETHEO are described. Section 6 reports on the performance gains achieved by subgoal alternation. We conclude then with a summary of the advantages of subgoal alternation and we describe our future work.

2 Model Elimination and Connection Tableaux

Historically, model elimination was introduced as a two-sorted variant of resolution (see, e.g., [Lov78]). Proof-theoretically, however, model elimination can also be viewed as a specialized tableau method, the so-called connection tableau calculus [LSBB92, LMG94]. A connection tableau for a set of clauses is generated by first applying the start rule and then repeatedly applying either the reduction or the extension rule. Employment of the start rule means selecting a clause from a set of possible start clauses and attaching its literals to the root node; the start rule is the standard tableau expansion rule. The reduction rule permits

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2 One major argument for this interpretation is that, when viewed as a tableau method, model elimination (in its weak variant) is cut-free.