1. MATHEMATICAL THEORY OF EVIDENCE
ON SPOHN'S THEORY OF EPISTEMIC BELIEFS

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ABSTRACT

This paper is about Spohn's theory of epistemic beliefs. The main ingredients of Spohn's theory are (i) a functional representation of an epistemic state called a disbelief function, and (ii) a rule for revising this function in light of new information. The main contribution of this paper is as follows. First, we provide a new axiomatic definition of an epistemic state and study some of its properties. Second, we state a rule for combining disbelief functions that is mathematically equivalent to Spohn's belief revision rule. Whereas Spohn's rule is defined in terms of the initial epistemic state and some features of the final epistemic state, the rule of combination is defined in terms of the initial epistemic state and the incremental epistemic state representing the information gained. Third, we state a rule of subtraction that allows one to recover the addendum epistemic state from the initial and final epistemic states. Fourth, we study some properties of our rule of combination. One distinct advantage of our rule of combination is that besides belief revision, it can also be used to describe an initial epistemic state for many variables when this information is provided in the form of several independent epistemic states each involving a small number of variables. Another advantage of our reformulation is that we are able to demonstrate that Spohn's theory of epistemic beliefs shares the essential abstract features of probability theory and the Dempster-Shafer theory of belief functions. One implication of this is that we have a ready-made algorithm for propagating disbelief functions using only local computation.

KEY WORDS: Spohn's theory, consistent epistemic state, content of an epistemic state, disbelief function, Spohnian belief function, Spohn's rules for belief revision, λ-conditionalization, rule of combination for disbelief functions, rule of subtraction for disbelief functions, axioms for local computation of marginals.

1. INTRODUCTION

This paper is about Spohn's theory of epistemic beliefs (Spohn 1988, 1990). Spohn's theory is an elegant, simple and powerful calculus designed to represent and reason with plain human beliefs. The main ingredients of Spohn's theory are (i) a functional representation of an epistemic state called a natural (or ordinal) conditional function, and (ii) a rule for revising this function in light of new information. Since the values of a natural conditional function represent degrees of disbelief, we call such a function a disbelief function. Like a probability distribution function, a disbelief function for a variable is completely specified by its values for the singleton subsets of configurations of the variable. Spohn (1990, pp. 152-154) has interpreted the values of a disbelief function as infinitesimal probabilities (see also (Pearl 1989)). Smets (private communication) and Dubois and Prade (1990) have pointed out that a disbelief function can be interpreted as the negative of the logarithm of a possibility function as studied by, e.g., Zadeh (1978) and Dubois and Prade (1988).

The main contribution of this paper is as follows. First, we provide an axiomatic definition of a consistent epistemic state. Some of the axioms we propose are different from the ones proposed by Spohn. Our axioms are a little easier to understand, but we show that the two sets of axioms are mathematically equivalent. These axioms are also found in (Gardenfors, 1988).

Second, we state a rule of combination for disbelief functions that is mathematically equivalent to Spohn's belief revision rule. Whereas Spohn's rule is defined in terms of the initial epistemic state and some features of the final epistemic state, the rule of combination is defined in terms of the initial epistemic state and the incremental epistemic state representing the information gained. The rule of combination for disbelief functions is pointwise addition.

Third, we state a rule of subtraction that allows one to always recover the addendum epistemic state from the initial and final epistemic states. This rule is useful in cases where the information gained is ex-