First, this paper investigates a model of the database with fuzzy information and generalizes a class of fuzzy indiscernibility relations from the model. Next, this paper is focused on algebraic analysis of the fuzzy indiscernibility, i.e., defining an algebraic structure based on the fuzzy indiscernibility; showing the representation theorem and the center of a given algebra.

**Keywords** Fuzzy information system, fuzzy indiscernibility, representation theorem, algebra of fuzzy indiscernibility, center of an algebra.

1. INTRODUCTION

In 1980, Pawlak [9] proposed a mathematical framework, i.e., the so called information system, to formalize the knowledge representation. Later, several researchers introduced fuzzy concepts into the information system based on the work of Pawlak, e.g., fuzzy rough sets [3, 4, 6], rough fuzzy sets [3] and a fuzzy modal logic [7]. In addition, the notion of fuzziness in indiscernibility was suggested by Farinas del Cerro and Prade [4].

The motivation of this approach is stimulated by the difficulty of checking the indiscernibility of objects when attributes of the objects are ambiguous and imprecise. For example, assume that person p1 is very tall and person p2 is extremely tall; the indiscernibility of p1 and p2 with respect to the height can be described by not only "yes" and "no", but also "fairly" and "hardly" .... To give such a description, we adopt a class of fuzzy relations, called fuzzy indiscernibility relations, and model them by a class of algebraic structures.

This paper is devoted to presenting a model of a database with fuzzy information which we call a fuzzy information system in correlation with Pawlak's information system; defining distinctively the notion of fuzzy indiscernibility of objects in the fuzzy information system and to giving an algebraic approach to it.
2. BASIC NOTIONS

To discuss the above mentioned problem, we define the following notions.

The following operations on the interval \([0,1]\) of real numbers from 0 to 1 follow from [2], [5]:

For \(p, q \in [0,1]\),

\[
p \wedge q = \max(0, p+q-1),
\]

\[
p \vee q = \min(1, p+q),
\]

\[
p \rightarrow q = \min(1, 1-p+q).
\]

\((p \rightarrow q) \wedge (q \rightarrow p)\) is abbreviated to \(p \rightarrow q\).

**Lemma 2.1.**

1) \(p \vee q = q \vee p; p \wedge q = q \wedge p\).
2) \((p \vee q) \wedge r = p \vee (q \wedge r); (p \wedge q) \wedge r = p \wedge (q \wedge r)\).
3) \(p \vee q = (1-p) \rightarrow q\).
4) \((p \rightarrow q) \rightarrow q = \min(p, q)\).
5) \(1-((1-p) \rightarrow (1-q)) \rightarrow (1-q)) = \max(p, q)\).
6) \(p \leftrightarrow q = 1-|p-q|\).

**Proof.** The proofs of 1)~5) are immediate.

\[
6) p \leftrightarrow q = \max(0, \min(1, 1-p+q) + \min(1, 1-q+p) - 1)
\]

\[
= \min(\min(1, 1-p+q), \min(1, 1-q+p))
\]

\[
= \min(1-p+q, 1-q+p)
\]

\[
= 1-|p-q|.
\]

A fuzzy information system is the quadruple \(I=(OB, FA, [0,1])\) where \(OB\) is an ordinary set of objects; \(FA\) is a finite set of fuzzy attributes of the objects, e.g., tall; \(g: OB \times FA \rightarrow [0,1]\) is a mapping which describes fuzzy predicates (e.g., an object has some fuzzy attribute ) by the real numbers in \([0,1]\). In fact, a fuzzy information system can be regarded as a database with fuzzy information. Intuitively, any fuzzy information system can be illustrated by the following table:

<table>
<thead>
<tr>
<th>(a_1)</th>
<th>(a_2)</th>
<th>\ldots</th>
<th>(a_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(o_1)</td>
<td>(v_{1,1})</td>
<td>(v_{1,2})</td>
<td>\ldots</td>
</tr>
<tr>
<td>(o_2)</td>
<td>(v_{2,1})</td>
<td>(v_{2,2})</td>
<td>\ldots</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\ldots</td>
</tr>
<tr>
<td>(o_m)</td>
<td>(v_{m,1})</td>
<td>(v_{m,2})</td>
<td>\ldots</td>
</tr>
</tbody>
</table>

That is, in the considered fuzzy information system, \(OB = \{o_1, o_2, \ldots, o_m, \ldots\}\), \(FA = \{a_1, a_2, \ldots, a_n\}\) and \(g(o_i, a_j) = v_{i,j} \in [0,1]\) for \(i = 1, 2, \ldots, m, \ldots, j = 1, 2, \ldots, n\), where \(m\) and \(n\) are finite; usually, the value \(v_{i,j}\) is used to represent the grade of fuzziness of the statement: the object \(o_i\) has the fuzzy attribute \(a_j\).

Given a fuzzy information system \(I\), the presentation of fuzzy indiscernibility in \(I\) is provided as follows: for \(A \subseteq FA\), a fuzzy indiscernibility relation \(\mathcal{A}(X)\) is a fuzzy subset of \(X \times X \subseteq OB \times OB\) characterized by the membership function \(\mu\):

\[
\text{if } A \neq \emptyset, \text{ then } \mu_{\mathcal{A}(X)}(o_1, o_2) = \bigwedge_{a \in A} (g(o_1, a) \leftrightarrow g(o_2, a)) \text{ for } o_1, o_2 \in X,
\]