Abstract

In this paper, it is considered the concept of conditioning for a family of possible probability distributions. First, the most used definitions are reviewed, in particular, Dempster conditioning, and upper-lower probabilities conditioning. It is shown that the former has a tendency to be too informative, and the last, by the contrary, too uninformative. Another definitions are also considered, as weak and strong conditioning. After, a new concept of conditional information is introduced. It is based on lower-upper probabilities definition, but introduces an estimation of the true probability distribution, by a method analogous to statistical maximum likelihood.

Finally, it is deduced a Bayes formula in which there is no 'a priori' information. This formula is used to combine informations from different sources and its behavior is compared with Dempster formula of combining informations. It is shown that our approach is compatible with operations with fuzzy sets.

Keywords

Theory of Evidence, conditioning, combining informations, Bayes rule.

1. INTRODUCTION

First of all, we are going to consider different kinds of informations that we may have about a variable X taking values on a finite set U. If the value of X is not determined, a probabilistic information may be available, that is a probability measure

\[ P: \mathbb{P}(U) \rightarrow [0,1], \]

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or its corresponding probability distribution \( p \).

Our point of view about this kind of informations is objective. That is, \( P(A) \) is interpreted as the limit of relative frequencies, considering an infinite sequence of independent values of variable \( X \) in the same conditions.

If we do not have an information as precise as a probability, we may have a set of possible probability distributions, \( \mathcal{E} = \{ P_1, \ldots, P_n \} \). Usually, this set will be a convex set of probabilities \( \mathcal{E} = \left\{ \sum_{i=1}^{n} \alpha_i P_i / \sum_{i=1}^{n} \alpha_i = 1 \right\} \), where \( P_1, \ldots, P_n \) are given probability measures.

With a set of probability measures, \( \mathcal{E} = \{ P_1, \ldots, P_n \} \), we may associate a convex set of probabilities, precisely its convex hull, \( \mathcal{E} = \left\{ \sum_{i=1}^{n} \alpha_i P_i / \sum_{i=1}^{n} \alpha_i = 1 \right\} \). \( \mathcal{E} \) and \( \mathcal{E} \) can be considered for the same experiment but with different interpretations. For example, assume that we have two urns, \( U_1 \) and \( U_2 \). \( U_1 \) has 99 red balls and one black ball. \( U_2 \) has one red ball and 99 black ones. If we pick up randomly a ball from one of the two urns, then for the ball color we have two possible probabilities, \( \mathcal{E} = \{ P_1, P_2 \} \), one for each urn. However, the experiment may be also considered as the selection of one urn and then picking up a ball. In this case, the frequencies of colors will be given by a probability \( \alpha P_1 + (1-\alpha)P_2 \). As \( \alpha \) is unknown, we have the convex set \( \mathcal{E} \) of possible probabilities. We will consider always the second interpretation. However, in the cases where two or more experiments are done with the same unknown probability it will be necessary to distinguish the two situations by introducing the set of possible probabilities as a parameter.

Another important representation of uncertainty are probability envelopes (or lower and upper probabilities). A probability envelope is a pair of ordered fuzzy measures, \( (1, u) \), (see [5]), such that there exists a family, \( \mathcal{P} \), of probability measures verifying

\[
\begin{align*}
  l(A) &= \inf_{P \in \mathcal{P}} \{ P(A) \} \\
  u(A) &= \sup_{P \in \mathcal{P}} \{ P(A) \}.
\end{align*}
\]

It is clear that, given a set of probabilities, \( \mathcal{E} \), we may associate a probability envelope with it. However, a probability envelope, \( (1, u) \), may be defined from different sets of probabilities. But in every case, there is always a maximal family, given by

\[
\mathcal{P} = \{ P / l(A) \leq P(A) \leq u(A), \forall A \subseteq U \}.
\]

If \( \mathcal{E} \) is a set of probabilities and we calculate the associated envelope, \( (1, u) \), this envelope is equivalent to a maximal family \( \mathcal{P} \). In this situation, \( \mathcal{E} \subseteq \mathcal{P} \) is always verified. In this sense, we can say that if we transform a set \( \mathcal{E} \) on an envelope, then some information is lost (there are more probability measures being possible).

Finally, a pair of belief-plausibility measures \( (\text{Bel}, \text{Pl}) \) will be a probability envelope such that there exists a function

\[
  m: \mathcal{P}(U) \longrightarrow [0,1]
\]

called basic probability assignment and such that