Expressive Completeness of LTrL on Finite Traces: An Algebraic Proof

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Abstract. Very recently a new temporal logic, for Mazurkiewicz traces, denoted LTrL, has been defined by Thiagarajan and Walukiewicz [15]. They have shown that this logic is equal in expressive power to the first order theory of finite and infinite traces thus filling a prominent gap in the theory.

We propose in this paper a entirely new, algebraic, proof of this result in the case of finite traces only. Our proof generalizes Cohen, Perrin and Pin’s work on finite sequences [2], using as a basic tool a new extension of the wreath product principle on traces [7].

As a major consequence of our proof we show that, when dealing with finite traces only, no past modality is necessary to obtain a expressively complete logic. Precisely, we prove that the logic LTrL_red, obtained from LTrL by not using the past modularity, has the same expressive power as the first order theory on finite traces.

Topics: logic in computer science, automata and formal languages, theory of parallel and distributed computation

1 Introduction

A run of a distributed system can be viewed, in many settings, as a partial order between the events of the system. Two events are ordered if and only if their executions depend causally one of the other. The partial orders that arise in this fashion, are frequently Mazurkiewicz traces [9,3]. A major interest of this model lies on the fact that a trace can be seen either as a labelled ordered graph expressing directly the partial order or as an equivalence class of sequences, each of them representing a linearization of the partial order.

Through this second approach, properties of distributed runs can be described with LTL, the propositional temporal logic of linear time [11]. If the required property is insensitive to the choice of linearizations (i.e. either all the members of an equivalence class satisfy the formula or none), it suffices to check the property for just one element of each equivalence class. From a practical point of
view, many of the so-called partial order reductions techniques in the verification process are based on this principle [6,10,17].

In a natural way, and in order to exploit directly the partial order underlying to a trace, a good amount of research have focused on developing temporal logics that can be directly interpreted over traces seen as labelled partially ordered graphs (rather than as sets of sequences). After the first attempt of Thiagarajan [14], several such logics have been proposed in the literature [4,1,12]. Until recently, none of them was completely satisfactory since none was expressively complete, i.e. had the same expressive power as the first order theory of finite and infinite traces. This was a gap in the generalisation of the theory of sequences to the theory of traces.

Very recently Thiagarajan and Walukiewicz [15] have defined a new temporal logic patterned on $LTL$, and denoted by $LTrL$, with precisely this expressive power. This logic uses classical $Next$ and $Until$ operations but also a restricted past modality making thus an annoying difference with $LTL$. Past modalities were also used extensively in the logic $TPLO$, due to Ebinger [4], which is expressively complete when interpreted on finite traces, only. Therefore past modalities can seem necessary to get expressively complete logics on traces.

We propose in this paper an entirely new proof of the completeness of $LTrL$ when interpreted on finite traces only. As a major consequence, we show that, when dealing with finite traces only, no past modality is necessary to obtain a logic expressively complete. Precisely, we prove that the logic $LTrL_{red}$, obtained from $LTrL$ by not using the past modality, has the same expressive power as the first order theory $FO(<)$ of finite traces.

The main difficulty is to show that any trace language $FO(<)$-definable is also $LTrL$-definable. To this purpose, our proof generalizes Cohen, Perrin and Pin’s results on finite sequences [2]. More precisely, it is known that $FO(<)$-definable trace languages coincide with aperiodic trace languages [16,8,4], that is trace languages recognized by aperiodic monoids. Note that these two results form an nice extension to traces to the famous Schützenberger’s theorem. Thus, we first study the aperiodic trace languages. Using as a basic tool a recent extension of the wreath product principle to traces [7], we propose a decomposition theorem of aperiodic trace languages. Thus extending the $PTL$ logic of [2], we introduce as an intermediary step a completely past oriented logic on traces, denoted by $PTrL$ and we prove that any aperiodic trace languages is $PTrL$-definable. We conclude easily our proof using the fact that a trace language is $PTrL$-definable if and only if its mirror is $LTrL_{red}$-definable and that aperiodic trace languages are closed under the mirror operator.