A LOCAL THEORY OF LINEAR SYSTEMS
WITH NONCOMMENSURATE TIME DELAYS

by

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Abstract

Stability and feedback control of linear neutral (and retarded) time-delay systems with one or more noncommensurate time delays is studied. The theory is based on a pointwise or local approach involving polynomial and rational functions in the complex variables $s, z_1, z_2, \ldots, z_q$, with $s$ evaluated at points in the right-half plane and the $z_i$ evaluated at points in the unit disc. In terms of this framework, an algebraic notion of stability, called pointwise stability, is defined and studied. Necessary and sufficient conditions are then given for the existence of a stabilizing dynamic output feedback compensator. The problem of stabilization using nondynamic state feedback is also briefly considered in the case when the system's input matrix has constant rank.

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1. Introduction

In this paper we continue the study of linear time-invariant continuous-time systems with time delays given by state equations whose coefficients belong to a ring of delay operators in one or more noncommensurate time delays. The delay-operator approach to retarded time-delay systems, whose origin can be traced back to KAMEN [1974, 1975] and WILLIAMS and ZAKIAN [1977], is based on an isomorphism between the ring of polynomial delay operators in \( q \) noncommensurate delays and the ring \( \mathbb{R} [ z_1, z_2, \ldots, z_q ] \) of polynomials in the symbols \( z_1, z_2, \ldots, z_q \) with coefficients in the reals \( \mathbb{R} \). Although the retarded case is of primary interest to us, our framework includes a large class of neutral time-delay systems, which is studied in terms of a subring of the field \( \mathbb{R}( z_1, z_2, \ldots, z_q ) \) of rational functions in the \( z_i \). For the case \( q = 1 \) (the commensurate-delay case), this ring approach to neutral systems was introduced by SONTAG [1976], and has been studied by SPONG and TARN [1982a, 1982b]. The general case \( q \geq 1 \) has been studied by SPONG [1981, 1982], BYRNES, SPONG, and TARN [1982], and KAMEN, KHARGONEKAR, and TANNENBAUM [1982].

After formulating the ring framework in the first part of Section 2, we consider a notion of pointwise stability specified in terms of the location of the zeros of a polynomial in \( s \) with coefficients in a subring of \( \mathbb{R}( z_1, z_2, \ldots, z_q ) \). For retarded time-delay systems with commensurate delays, this notion of stability was introduced and studied by KAMEN [1980, 1982, 1983]. (For results on pointwise stability in the neutral case, see JURY and MANSOUR [1982] and GUIVER and BOSE [1981].) In Section 2 pointwise stability is characterized by several equivalent conditions; in particular, it is shown that pointwise stability is generically equivalent to uniform asymptotic stability independent of delay.

In Section 3 we study the problem of stabilization via dynamic output feedback. We say that a given time-delay system is regulable if there is a dynamic output feedback system with pure delays such that the closed-loop system is internally pointwise stable. Regulability is a sufficient condition for the existence of a dynamic output feedback system with pure delays such that the closed-loop system is uniformly asymptotically stable independent of delay. It should be noted that the problem of determining when there is a dynamic output feedback system with pure delays such that the closed-loop system is uniformly asymptotically stable independent of delay is still very much open, although there are results from the functional-analytical theory of time-delay systems which guarantee the existence of stabilizing compensators containing in general both pure and distributed delays (see PANDOLFI [1975]). The major new result in Section 3 is the derivation of necessary and sufficient conditions for regulability expressed in terms of two spectral conditions involving the coefficient matrices of the given state-equation representation. It is also shown in Section 3 that if the given system is a retarded time-delay system, then regulability implies that there is a stabilizing feedback compensator which is also a retarded time-delay system.