CHARACTERIZATION OF REDUNDANCY IN SPATIAL CLOSED KINEMATIC CHAINS

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ABSTRACT

The force allocation problem in closed kinematic chains is underdetermined and the available degrees of freedom can be used to extremize a desired objective function. This generally leads to complicated nonlinear optimization problems. The solution of these problems can be greatly simplified if the redundancy is characterized in the simplest possible form. This paper gives strategies that lead to efficient algorithms to evaluate the null space of the linear set of force allocation equations. The approach exploits the geometry and physics in the problem rather than looking at numerical solutions. The general problem of \( n \) contact points of a kinematic chain in three dimensions is considered. Further, degenerate situations occurring due to the presence of several points along a line, or on a plane, are also addressed. Numerical examples are considered to illustrate the use of these algorithms.

1. INTRODUCTION

The force allocation problem has been studied with respect to two basic types of kinematic chains. The first type includes multi-fingered hands grasping an object. Multi-fingered hands have been studied in literature [Mason and Salisbury 1985], [Coe 1989]. The optimization of the friction conditions at the contact points has been studied for three fingered grasps [Demmel and Laferriere 1989], [Ji and Roth 1988], [Mukherjee 1991]. The second type of kinematic chains involves mobile robotic vehicles. Several mobile vehicle systems have been studied [Hirose 1984], [Raibert 1986], [Pugh et al. 1990]. Linear programming sub-optimal solutions to the force allocation optimization problems are available [Cheng and Orin 1989]. The characteristics of the minimum norm pseudo-inverse solution to the force allocation problem are found in the reference [Kamar and Waldron 1988]. This reference also addresses the null space force field known as the interaction force field. A typical multi-fingered grasping situation is shown in Figure 2 and a mobile robotic platform interacting with its environment is shown in Figure 3. The force allocation problem in both these situations is very similar. Hence, a large portion of this study is common to both kinds of situations.

2. BASIC DEFINITIONS AND NOTATION

A rigid body interacting with its environment is shown in Figure 1. There are \( n \) contact points \( C_i \). The contact force vector at the contact point \( C_i \) is \( F_i \) and the surface normal at \( C_i \) is \( N_i \). \( F_B \) and \( T_B \) represent the body force and moment respectively (inertial and gravitational).

\[
F_i = [F_{ix}, F_{iy}, F_{iz}]^T, \quad C_i = [x_i, y_i, z_i]^T, \quad Q = [F_B^T, T_B^T]^T, \quad F = [F_{ix}, F_{iy}, F_{iz}, .., F_{nx}, F_{ny}, F_{nz}]^T
\]

The rigid body dynamic equations are:
\[ [G] F = Q \]  \hspace{1cm} (1)

\begin{align*}
[G] &= \begin{bmatrix} I_3 & I_3 & \cdots & I_3 \\ R_1 & R_2 & \cdots & R_3 \end{bmatrix}, \\
R_1 &= \begin{bmatrix} 0 & z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix}
\end{align*}

\( I_3 \) is the 3x3 identity matrix. \( [G] \) is a matrix purely dependent on the geometry of the contact points. For a given vector \( Q \), the force allocation problem involves solving Equation 1 for the contact force vector \( F \). Equation 1 represents six linear equations in 3n unknown force components. The rank of the matrix \( [G] \) is six unless all the n contact points are collinear. This special situation can be regarded as trivial because physical problems rarely involve n collinear points. Hence, throughout this discussion, rank of \( [G] \) is taken to be equal to six. The matrix \( [G] \) is 3nx6 matrix. If \( N(G) \) represents the null space of the \( [G] \), \( \dim(N(G)) = 3n - 6 \).

Kumar and Waldron [Kumar and Waldron 1988] addressed the redundancy in force allocation by defining two force fields; the \textit{equilibrating} force field and the \textit{interaction} force field. The equilibrating force field was shown to be the same as the pseudo-inverse solution to Equation 1, and this force field had no contribution from \( N(G) \). The interaction force field was shown to belong to \( N(G) \). An interaction force was defined as a vector of forces belonging to the solution space of Equation 1 that had two equal and opposite contact forces acting between two contact points along the line joining the two contact points. The interaction force field consists of all possible interaction forces among the n points. If \( F^* \) is the equilibrating force vector. Then,

\[ F^* = [G]^+ Q \]  \hspace{1cm} (2)

where \( [G]^+ \) is the pseudo-inverse of the non-square matrix \( [G] \) and
\[ [G]^+ = [G]^T ([G] [G]^T)^{-1} \]  \hspace{1cm} (3)

\( F^* \) can be obtained more efficiently by the use of the equilibrating force field [Kumar and Waldron 1988]. Let the vector joining a set of two points of contact, i and j, be \( \mathbf{v} \) (\( \mathbf{v} \) belongs to \( \mathbb{R}^3 \)). Then the interaction force between these two points, for any scalar \( k \), is of the form

\[ F_{ij} = k [0^T, 0^T, \cdots, v^T, 0^T, \cdots, -v^T, \cdots, 0^T]^T \]

Here \( F_{ij} \) is a 3nx1 vector made of n vectors in \( \mathbb{R}^3 \) and only the ith and jth vectors are non-zero.

There are exactly \( n(n-1) \) interaction forces that can be obtained for a set of n points. The interaction force field does not exist for the case of n equal to 1. When n takes the value of two or three, \( n(n-1) \) is equal to 3n - 6 and it can be easily shown that \( n(n-1) \) interaction force vectors form a basis for \( N(G) \). When n takes values greater than three obtaining a basis for \( N(G) \) is a non-trivial problem. For a situation involving four contact points, \( n(n-1) \) is equal to 3n - 6. However, as discussed in later sections, in certain situations, the interaction force field is a subspace of \( N(G) \). For n greater than four, \( n(n-1) \) is greater than 3n - 6. Thus the choice of a basis requires an efficient selection process that leads to simple basis vectors. Also for any value of n greater than three there always exist degenerate situations in which the interaction force field (even though \( n(n-1) \) is much greater than 3n - 6) has a dimension lower than that of the N(G). Some fundamental results leading to the characterization of \( N[G] \) are given in the next section.

### 3. FUNDAMENTAL RESULTS

This section considers a finite set of contact points and investigates the nature of the interaction force field in detail. Two types of distribution of the contact points are considered. The first type includes contact points which are spatially distributed and the second type consists of a set of contact points lying on a plane. A collinear set of points are not considered as this leads to a singular situation where the rank of \( [G] \) is less than six.