QUANTUM CORRECTIONS FOR THE MAGNETIC SOLITONS IN
VARIOUS HEISENBERG MODELS

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Introduction

From the early days of quantum theory of magnetism, i.e. from the
times of Heisenberg ferromagnet, two approaches appeared: one treating
spins (magnetic moments) as the operators of the angular momentum and
the other one trying to represent them by Bose-operators. Both
approaches have their pro's and contra's, but today one can read in most
of the textbooks that boson representations "work" well at low
temperatures, while for higher temperatures one has to take into account
proper nature of spins.

When the soliton theory was first applied to the Heisenberg
ferromagnet, the spins were treated as classical vectors [1]. Only
later, various quantum corrections were looked for. The question that
arose naturally was: if boson approach can describe spins for some other
purposes, can it work here? The most suitable representation was
Holstein-Primakoff (HP) representation [2] for at least two reasons: it
is not limited to low-temperatures in the most general form and its
validity becomes better for higher spins ($S >> 1$) which agrees well with
the classical limit ($S \rightarrow \infty$, $\hbar \rightarrow 0$, $Sh=S_c$).

For the system with the ground state of all spins pointing "up", HP
representation takes the form:

$$\hat{S}_n^z = S - \hat{B}_n^+ \hat{B}_n$$

$$\hat{S}_n^+ = \sqrt{S} \sqrt{1 - \frac{\hat{B}_n^+ \hat{B}_n}{2S}} \hat{B}_n \quad \hat{S}_n^- = (\hat{S}_n^+)^*$$

where $\{B_n^+, B_n\}$ are Bose-operators satisfying commutation relations

$$[\hat{B}_n, \hat{B}_m^+] = \delta_{n,m} \quad [\hat{B}_n^+, \hat{B}_m^+] = [\hat{B}_n, \hat{B}_m] = 0$$

(We must notice that this representation is valid for the system $\hbar=1$,
so in order to perform the classical transition correctly, we shall
introduce factors of $\hbar$ into the Hamiltonian.)

The general procedure for the application of boson representations
in the soliton theory is rather simple and consists in the following
steps:
a) expressing spin operators in terms of Bose-operators ("bosonisation");

b) averaging the Hamiltonian over Glauber's coherent states (CS) [3]:

\[
| \hat{B}_n \alpha_n \rangle = \alpha_n | \alpha_n \rangle
\]  

(2)

c) treating the averaged Hamiltonian as the classical Hamilton's function of the system with \( \alpha \) and \( \text{i} \alpha^* \) as conjugated variables [4];

d) expressing the results in terms of physical relevant quantities (energy, momentum, magnetization).

How it occurred that such simple procedure failed to give reasonable results in the earliest attempts can be explained by the fact that the application of CS demands the ordered product of operators, which is not easy to obtain working with the square root in HP representation (1).

Quantum corrections for spin operators

We shall now present our approach which produces correct results in the classical limit and also allows the calculation of quantum corrections up to any order of \( 1/S \). This will be illustrated with the correction of order \( 1/S \). Since we are interested in the classical limit, we suppose \( S \) to be large enough that we can perform the expansion of the form [5]:

\[
\left( 1 - \frac{\hat{B}_j \hat{B}_j}{2S} \right)^{1/2} = \sum_{k=0}^{\infty} \left( \frac{1}{2} \right)^k \frac{\hat{B}_j \hat{B}_j}{S^k}
\]  

(3)

Putting \( \left( \hat{B}_j \hat{B}_j \right)^k \) into the normal ordered form, it can be written generally as

\[
\left( \hat{B}_j \hat{B}_j \right)^k = \hat{B}_j^k \hat{B}_j + a_k^{k} \hat{B}_j^{k-1} \hat{B}_j^{-1} + a_k^{k} \hat{B}_j^{k-2} \hat{B}_j^{-2} + \ldots + a_k^{k} \hat{B}_j \hat{B}_j + a_k^{k} \hat{B}_j \hat{B}_j
\]  

(4)

It can be seen by inspection that \( a_k^k = a_1^k = 1 \). This expression shows that for any order \( \hat{B}_j^l \hat{B}_j^l \) there appear also the contributions arising from \( \left( \hat{B}_j \hat{B}_j \right)^p \) for any \( p > l \). Yet, expression (3) shows that such contributions are divided with \( S^p \) (\( p > l \)) and these terms are lost in the classical limit.

We can determine these terms, for example, by calculating \( \left( \hat{B}_j \hat{B}_j \right)^{k+1} \).