Symbolic Computing Aided Design of Nonlinear PID Controllers

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Abstract. In this paper we introduce a symbolic computing tool, denoted by NLPID in the sequel, for the automatic design of linear and nonlinear PID controllers for nth order nonlinear control systems. The nonlinear design algorithm is based upon Rugh's Extended Linearization Technique, and it was implemented using Mathematica® as symbolic computing platform. At its present stage of development NLPID uses Ziegler-Nichols tables to synthesize linear PID controllers, and therefore its ability to deal with first and second order plants could be limited.

Keywords: Nonlinear PID Controllers, Jacobian and Extended Linearization, Symbolic Computing.

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1 Introduction

NonLinear Control Systems (NLCS) are dynamical systems defined through (i) a state equation, i.e., an ordinary differential equation:
where $f$ is at least Lipschitz continuous to assure the initial value problem has a unique solution, and (ii) an algebraic output equation:

$$y = h(x), \ y \in \mathbb{R}^m. \quad (1-b)$$

An operating point of the NLCS (1) is a point $(U, X(U), Y(U))$ in $\mathbb{R}^k \times \mathbb{R}^n \times \mathbb{R}^m$ such that $f(X(U), U) = 0$ and $Y(U) = h(X(U))$. We will assume in the sequel that $k = m = 1$, and that the output function $h$ is continuous.

One of the main problems of control theory is the stabilization problem, which might be mathematically formulated as follows: Does it exist a control law $u$ capable of asymptotically stabilize the NLCS (1) to a specified operating point $(U, X(U), Y(U))$? From the engineering point of view, however, the actual question is: Is it possible to exhibit, by way of a designing algorithm, a control law $u$ that asymptotically stabilized the NLCS (1) to a desired operating point $(U, X(U), Y(U))$? The answer to this question is generically affirmative provided the usual controllability conditions hold, and that we think of stabilization only in the restricted, local sense, i.e., of stabilization in small enough neighborhoods of the desired operating point.

Local stabilization of control systems can be approached through different control strategies: state feedback controllers, PID controllers, lead-lag compensators, amongst others. In this paper we will focus our attention unto the PID-family of control strategies, and our main purpose is to develop a symbolic computing tool for the automatic design of nonlinear PID controllers by using the extended linearization technique.

2. The Jacobian Linearization Method

The most frequently used local stabilization strategy is Jacobian Linearization Method (JLM), according to which we must firstly stabilize the linearization of the NLCS (1) around the desired operating point $(U, X(U), Y(U))$ with a linear control law $u = G(x, y)$, to proceed, secondly, to locally stabilize the NLCS (1) with the very same linear controller. After the Hartman-Grobman theorem (Hartman 1982, Perko 1991) the dynamic of the closed loop linearized system:

$$\xi' = D_x f(X(U), U)\xi + D_u f(X(U), U)u = A(U)\xi + B(U)u \quad (2-a)$$