Abstract

We review our model of chaotic neural networks with chaotic dynamics and apply it to associative memory. We demonstrate that dynamical associative memory can be realized with the chaotic neural network.

1 Introduction

It is widely recognized that chaotic phenomena are observed in many fields of science and technology[1]. It is also reported that there exists chaotic dynamics not only in neurons but in neural networks or brains[2],[3].

In this paper, we apply a chaotic neural network to associative memory and analyze its dynamical behavior. In order to confirm chaotic dynamics in associative dynamics, we calculate the Lyapunov spectrum, the temporal changes of the distance between the output of the chaotic neural network and the stored patterns, and the local divergence rate.

2 The model of a chaotic neuron and its network

2.1 A Chaotic Neuron Model

A chaotic neuron model is obtained by modifying the discrete-time neuron models of Caianiello[4] and Nagumo and Sato[5]. The dynamics of a chaotic neuron with graded output and exponen-
tainly decaying refractoriness is described by the following equation [6],[7]:

\[ x(t + 1) = f[A(t) - \alpha \sum_{d=0}^{t} k_d g\{x(t - d)\} - \theta] \]  

(1)

where \( x(t) \) is the output which takes an analog value between 0 and 1; \( A(t) \) is an externally applied input; \( f \) is the output function; \( g \) is the refractory function; \( \alpha, k, \) and \( \theta \) are the refractory scaling parameter, the refractory decay parameter, and the threshold, respectively.

Defining an internal state \( y(t+1) \) of the neuron as

\[ y(t + 1) = A(t) - \alpha \sum_{d=0}^{t} k_d g\{x(t - d)\} - \theta, \]  

(2)

we can get reduced difference equations [7]:

\[ y(t + 1) = k y(t) - \alpha g[f\{y(t)\}] + a(t), \]  

(3)

\[ x(t + 1) = f\{y(t + 1)\} \]  

(4)

where \( a(t) = A(t) - kA(t - 1) - \theta(1 - k) \).

2.2 The model of a chaotic neural network

We can design the model of a chaotic neural network by considering two kinds of input, i.e., feedback inputs from component neurons and externally applied inputs [7].

The dynamics of the \( i \)th chaotic neuron in a neural network composed of \( n \) chaotic neurons can be modeled as Eq.(5),

\[
\begin{align*}
x_i(t + 1) & = f \left[ \sum_{j=1}^{m} V_{ij} \sum_{d=0}^{t} k_d^A A_j(t - d) \\
& + \sum_{j=1}^{m} W_{ij} \sum_{d=0}^{t} k_d^W h\{x_j(t - d)\} - \alpha \sum_{d=0}^{t} k_d^g \{x_i(t - d)\} - \theta_i \right]
\end{align*}
\]

(5)

where \( x_i(t + 1) \) is the output of the \( i \)th chaotic neuron at the discrete time \( t + 1 \) and \( n \) is the number of the chaotic neurons in the network; \( A_j(t - d) \) is the strength of the \( j \)th externally applied input at the discrete time \( t - d \) and \( m \) is the number of the externally applied inputs; \( V_{ij} \) is the connection weight from the \( j \)th externally applied input to the \( i \)th chaotic neuron; \( W_{ij} \) is the connection weight from the \( j \)th chaotic neuron to the \( i \)th chaotic neuron; \( f \) is the continuous output function; \( h \) is the transfer function of the axon for the propagating action potentials; \( g \) is the refractory function. \( \alpha \) is the scaling parameter and \( k_d^A, k_m^W, k_d^g \) are the decay parameters for the external inputs, the feedback inputs and the refractoriness, respectively.

Defining three terms in the parenthesis of Eq.(5) by \( \xi_i, \eta_i \) and \( \zeta_i \) :

\[ \xi_i(t + 1) = \sum_{j=1}^{m} V_{ij} \sum_{d=0}^{t} k_d^A A_j(t - d), \]  

(6)